TENSOR-BASED CHANNEL ESTIMATION FOR NON-REGENERATIVE TWO-WAY RELAYING NETWORKS WITH MULTIPLE RELAYS

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ABSTRACT

In this paper we investigate a two-way relaying network with multiple amplify-and-forward relays where both the user terminals (UTs) and the relays can have multiple antennas. To improve the system performance, the UTs need channel knowledge of all relevant channels. Therefore, we propose block component decomposition based channel estimation (BloCE) algorithms, which use the Tucker decomposition and the CANDECOMP/PARAFAC (CP) decomposition. The proposed methods are analytic, i.e., iterations are not required. We also derive the design criteria for the corresponding relay amplification matrices and the training sequences. Simulation results demonstrate the accuracy of the proposed channel estimation method.

Index Terms— Two-way relaying, amplify and forward, tensor decomposition, channel estimation.

1. INTRODUCTION

Two-way relaying (TWR) networks are important for future mobile communication systems since they can improve the network performance by extending the coverage and increasing the network capacity [1], [2]. Advanced transmit strategies can be designed to take advantage of available physical resources such as multiple antennas in relaying networks, e.g., [3], [4]. Moreover, these designs require channel knowledge at the transmitter and/or at the receiver. To estimate the channel, the typical way is to use matrix based solutions, e.g., [5]. Recently, research results in [6] and [7] show that a tensor-based channel estimation method for relaying networks can provide a better estimation accuracy, requires less training and leads to less ambiguities compared to the matrix based solution.

Tensor-based channel estimation methods have not been studied for two-way relaying networks with multiple multi-antenna amplify-and-forward (AF) relays and multi-antenna user terminals (UTs). Hence, this motivates us to study novel channel estimation solution via tensor decompositions. In contrast to [7], where each relay use a diagonal relay amplification matrix, in our work each relay use a diagonal relay amplification matrix, in our work each

will call them the block component decomposition based channel estimation (BloCE) framework. Moreover, we develop design rules for the relay amplification matrix at each relay and obtain a non-iterative channel estimation method, when the BloCE framework is used. Simulation results demonstrate the achievable channel estimation accuracy of the BloCE framework with or without structured least squares (SLS).

Notation: Upper-case bold-faced letters, lower-case bold-faced letters, and bold-faced calligraphic letters denote matrices, vectors, and tensors, respectively. The expectation, transpose, conjugate, Hermitian transpose, and Moore-Penrose pseudo inverse are denoted by $E\{\cdot\}$, $\{\cdot\}^T$, $\{\cdot\}^H$, $\{\cdot\}^*$, and $\{\cdot\}^+$, respectively. The $m$–by–$n$ identity matrix is $I_{mn}$. The $m$–by–$n$ matrix with all zero elements is $0_{m \times n}$. The Euclidean norm of a vector and the absolute value are denoted by $\|\cdot\|$ and $|\cdot|$, respectively. The Kronecker product is $\otimes$. The Khatri-Rao product is denoted by $\odot$, which is defined as the column-wise Kronecker product. The symbol $\oplus$ represents the partition-wise Khroneck product [8]. For example, let us define $A = [A_1, A_2, \ldots, A_K]$ and $B = [B_1, B_2, \ldots, B_K]$. Then $A \oplus B = [A_1 \otimes B_1, A_2 \otimes B_2, \ldots, A_K \otimes B_K]$. The vec operator stacks the columns of a matrix into a vector. The $\text{vec}_{M \times N}\{\cdot\}$ operator stands for the inverse function of vec. A block diagonal matrix is created by the operation $\text{blkdiag}\{A_i\}_{i=1}^{n}$ or $\text{blkdiag}\{A, B\}$. The rank of a matrix is denoted by rank\{$\cdot$\}. The $n$-mode product $\times_n$ and the $n$-mode unfolding of a tensor $A$, i.e., $[A]_{n\times n}$, are defined in the same way as in [6]. The notation $\mathcal{O}$ denotes a tensor with only zero elements.
II. SYSTEM MODEL

The scenario under investigation is shown in Fig. 1, where two multi-antenna UTs communicate with each other via the help of \( L \) multi-antenna relays. The \( k \)-th \( (k \in \{1, 2\}) \) UT has \( M_k \) antennas. For notational simplicity, each relay has \( M_R \) antennas. All the nodes are half-duplex. We assume that the channel is frequency flat and quasi-static block fading. The channel matrix from the \( k \)-th UT to the \( \ell \)-th relay is denoted as \( H_{k,\ell}^{(t)} \in C^{M_R \times M_k} \) \( (\ell \in \{1, \ldots, L\}) \). Furthermore, we assume \( H_{k,\ell}^{(t)} \) is a full rank matrix which implies that \( \text{rank}(H_{k,\ell}^{(t)}) = \min\{M_R, M_k\} \). We define \( M_0 = M_1 + M_2 \). In order to estimate the channel \( H_{k,\ell}^{(t)} \) at the UTs, \( \forall k \), and at the relays, \( \forall \ell \), the training phase is assumed to consist of \( N_R \) frames. In each frame, each UT transmits \( N_T \) training vectors to the relays. This will result in a total number of \( N_R \cdot N_T \) training vectors per UT. Define \( i = 1, \ldots, N_R \) and \( j = 1, \ldots, N_T \). In the \( i \)-th frame, each UT transmits \( N_T \) training vectors to the relays using \( 2 \cdot N_T \) slots. In the \( (2j - 1) \)-th slot time, the accumulated received signal at the \( \ell \)-th relay is given as

\[
r_{k,\ell}^{(t)} = \sum_{i=1}^{N_R} H_{k,\ell}^{(t)} x_{k,j} + n_{R,i,j}^{(t)} \in C^{M_R}
\]

where \( x_{k,j} \) is the \( j \)-th column of \( X_k \in C^{M_k \times N_P} \) such that \( X_k \) is the training matrix of the \( k \)-th UT. The training sequences \( x_{k,j} \), \( \forall k,j \), have unit norms. The vector \( n_{R,i,j}^{(t)} \) represents the zero-mean circularly symmetric complex Gaussian (ZMCSG) noise and \( E[n_{R,i,j}^{(t)}n_{R,i,j}^{(t)*}] = \sigma_n^2 I_{M_R} \).

In the \( (2j) \)-th time slot, the relay amplifies the received training vectors and forwards them to both UTs simultaneously. The signal transmitted by the \( \ell \)-th relay is expressed as

\[
p_{k,\ell}^{(t)} = G_{k,\ell}^{(t)} n_{R,i,j}^{(t)} \in C^{M_R}
\]

where \( G_{k,\ell}^{(t)} \in C^{M_R \times M_k} \) is the relay amplification matrix during the \( \ell \)-th frame. We assume that an ideal TDD system is used and channel reciprocity is valid for the uplink and the downlink channels between the UTs and the relays [9]. Let \( H_{k}^{(t)} = [H_{k,1}^{(t)} \ldots H_{k,L}^{(t)}] \in C^{M_R \times M_k} \) and \( \tilde{X}_k = [X_k^T \ X_{3-k}^T]^T \in C^{M_R \times N_P} \). Finally, the received signal at the \( k \)-th UT in the \( i \)-th frame can be calculated as

\[
Y_{k,i} = \sum_{\ell=1}^{L} H_{k,\ell}^{(t)\dagger} G_{k,\ell}^{(t)} X_{\ell,\ell} + N_{k,i} \in C^{M_k \times N_P}
\]

where \( N_{k,i} = \sum_{\ell=1}^{L} H_{k,\ell}^{(t)\dagger} G_{k,\ell}^{(t)} N_{R,i} + \tilde{N}_{k,i} \), denotes the effective noise, \( N_{R,i} = [n_{R,1}^{(t)} \ldots n_{R,L}^{(t)}] \) and \( \tilde{N}_{k,i} \) denotes the ZMCSG noise at the \( k \)-th UT with \( E[\text{vec}(\tilde{N}_{k,i})\text{vec}(\tilde{N}_{k,i})^H] = \sigma_n^2 I_{M_k \times N_P} \), \( \forall k,i \). The required relay amplification matrices and training sequences are summarized in Table I.

In the following we show how to design the relay amplification matrices \( G_{k,\ell}^{(t)} \), \( \forall k, \ell \), can be uniquely identified at each UT.

<table>
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<th>Table I.</th>
<th>Training requirements during ( N_R ) frames</th>
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<td>Relay amplification matrix @ ( \ell )-th relay</td>
<td>( G_1^{(t)} \ldots G_{1,N_R}^{(t)} )</td>
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<td>Training sequences UT 1</td>
<td>( X_1 \ldots X_{1,N_T} )</td>
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III. BLOCK COMPONENT DECOMPOSITION BASED CHANNEL ESTIMATION

Before we introduce the proposed channel estimation algorithm, we write the received signal at the \( k \)-th UT using a tensor representation. Let us define

\[
Y_k = [Y_{k,1} \ldots Y_{k,N_R}] \in C^{M_k \times N_T \times N_R}
\]

\[
G^{(t)} = [G_1^{(t)} \ldots G_{1,N_R}^{(t)}] \in C^{M_R \times M_k \times N_R}
\]

\[
N_k = [N_{k,1} \ldots N_{k,N_R}] \in C^{M_k \times N_T \times N_R}
\]

where \( \omega_k \) represents the concatenation of two tensors along the \( r \)-th mode [10]. Then we obtain the following relationship at the \( k \)-th UT after \( N_R \) frames.

\[
Y_k = \sum_{\ell=1}^{L} G^{(t)} \times_1 H_{k,\ell}^{(t)\dagger} \times_2 (H_{k,\ell}^{(t)} X_{\ell,\ell}^T + N_{k,\ell})
\]

Without the noise term \( N_k \), the tensor-based system model (4) fits to the BCD Type-2 model in [8]. Therefore, one could apply the ALS algorithm in [11] to calculate the BCD of \( Y_k \) such that noise distorted versions of \( G^{(t)} \), \( H_{k,\ell}^{(t)} \), and \( H_{k,\ell}^{(t)\dagger} \) are obtained for all \( \ell \). Unfortunately, the BCD Type-2 decomposition is only unique up to scaling and permutation, which is not sufficient for identifying the channels accurately. Since the tensors \( G^{(t)} \), whose \( i \)-th frontal slice represents the relay amplification matrix of the \( i \)-th relay in the \( i \)-th frame can be designed in advance, our a priori knowledge of \( G^{(t)} \) is used to reduce the ambiguity in channel estimation.

The tensors \( G^{(t)} \) can be decomposed using tensor decompositions. If the Tucker decomposition is used, the tensor \( G^{(t)} \) can be decomposed as

\[
G^{(t)} = S^{(t)} \times_1 U_1^{(t)} \times_2 U_2^{(t)} \times_3 U_3^{(t)},
\]

where \( S^{(t)} \in C^{M_k \times 1 \times 1} \), \( U_1^{(t)} \in C^{M_k \times 1 \times 1} \), \( U_2^{(t)} \in C^{M_R \times 1 \times 1} \), and \( U_3^{(t)} \in C^{N_R \times 1 \times 1} \), \( \forall \ell \). For notational simplicity, in the rest of the paper we consider only the case that \( I_{k,\ell} = I_1, I_{2,\ell} = I_2, \) and \( I_{3,\ell} = I_3, \forall \ell \). Inserting (5) into (4), we get

\[
Y_k \approx \sum_{\ell=1}^{L} S^{(t)} \times_1 U_1^{(t)} \times_2 U_2^{(t)} \times_3 U_3^{(t)}
\]

where \( U_1^{(t)} \), \( U_2^{(t)} \), and \( U_3^{(t)} \) are the \( \ell \)-th sub-matrices of \( U_{1,k} \), \( U_{2,k} \), and \( U_{3} \), respectively. We define

\[
U_{1,k} = [H_{k,1}^{(t)\dagger} U_1^{(1)} \ldots H_{k,L}^{(t)\dagger} U_1^{(L)}] \in C^{M_k \times L_1}
\]

\[
U_{2,k} = [X_1^T H_{k,1}^{(t)\dagger} U_2^{(1)} \ldots X_3^T H_{k,L}^{(t)\dagger} U_2^{(L)}] \in C^{N_k \times L_2}
\]

\[
U_{3} = [U_3^{(1)} \ldots U_3^{(L)}] \in C^{N_R \times L_3}
\]

\[
\tilde{S} = \text{blkdiag} \left\{ (S^{(t)})_{(3)} \right\}_{\ell=1}^{L} \in C^{L_3 \times L_1 \times L_2}
\]

Equation (6) holds only approximately due to the presence of the noise. It has the form of the BCD in rank-\((I_1, I_2, I_3)\) terms in [8]. Again, the algorithm and the uniqueness condition for the BCD in rank-\((I_1, I_2, I_3)\) terms could be used for obtaining the terms in (6) if \( I_1 \leq M_k, I_2 \leq M_R, \) and \( I_3 \leq N_T \). Nevertheless, to make a better use of our knowledge of \( G^{(t)} \) and to reduce the ambiguity in channel estimation, we devise algebraic (non-iterative)
channel estimation methods. For this purpose, we apply the 3-mode unfolding of $Y_k$ in [8], which satisfies

$$[Y_k](3) \approx U_3 \cdot \bar{S} \cdot \left(\hat{U}_{1,k} \circ \rho_p \hat{U}_{2,k}\right)^T.$$  

(7)

Depending on the construction of the relay amplification tensors $G^{(3)}$, we can distinguish between a Tucker decomposition based solution and a CP decomposition based solution. As both solutions stem from the 3-mode unfolding of the BCD in rank-$(I_1, I_2, I_3)$ terms, we refer to the proposed solutions as the block component decomposition based channel estimation (BloCE) framework.

III-A. Tucker decomposition based solution

Let $\bar{U} = \hat{U}_1 \cdot \bar{S} \in C^{N_R \times L_1 L_2}$. The first method requires that the multiplication by $\bar{U}$ must be inverted. To guarantee that the partition-wise Kronecker product in (7) is uniquely identified, we require that $L_1 I_1 \geq N_R \geq L_1 I_2$. Moreover, we can choose $\bar{U}$ such that it has orthogonal columns, i.e., $\bar{U}^H \bar{U}$ is a scaled identity matrix. This guarantees that the inversion step does not affect the statistics of the noise. It also avoids explicit matrix inversion. In practice, $\bar{U}_3$ and $\bar{S}$ are required for constructing the tensors $G^{(3)}$ instead of their product $U \bar{U}$. To calculate $\bar{U}_3$ and $\bar{S}$ from $\bar{U}$, we use the following fact

$$\bar{U} = \bar{U}_3 \cdot \bar{S} = \left[U^{(1)}_3 \left[S^{(1)}_3 \right] \ldots U^{(L)}_3 \left[S^{(L)}_3 \right]\right],$$

(8)

where each sub-matrix $U^{(l)}_3$ occupies $(I_1, I_2)$ columns of $\bar{U}$. Let $\bar{U}_3 \in C^{N_R \times L_1 L_2}$ denote the $\ell$-th sub-matrix of $\bar{U}$. We can determine $U^{(l)}_3$ and $\left[S^{(l)}_3\right]$ by using a matrix decomposition of $\bar{U}_3$. For example, the SVD $\bar{U}_3 = Q_3 \Sigma_3 W^{H}_3$. The matrices $U^{(l)}_3$ and $\left[S^{(l)}_3\right]$ are chosen to be $U^{(l)}_3 = Q_3$ and $\left[S^{(l)}_3\right] = \Sigma_3 W^{H}_3$, respectively. Note that uniqueness is not a relevant problem here.

After multiplying $\bar{U}^+$ to both side of equation (7), we obtain

$$\left(\bar{U}^+ \cdot [Y_k]_{(3)}\right)^T \approx \hat{U}_{1,k} \circ \rho_p \hat{U}_{2,k},$$

(9)

where $\bar{U}^+$ is the pseudo-inverse of $\bar{U}$. Then the compound matrix $\left(\bar{U}^+ \cdot [Y_k]_{(3)}\right)$ is split into sub-matrices where the $\ell$-th sub-matrix is defined as $U^{(l)}_3$ and has $(M_k \cdot M_k)$ rows and $(I_1 \cdot I_2)$ columns. Equation (9) implies that

$$\left[U^{(l)} \cdot [Y_k]_{(3)}\right] = \Sigma_3 W^{H}_3.$$  

(10)

According to [12], the Kronecker-product in (10) can be inverted up to one scaling ambiguity using Algorithm 1. That means we can find matrices $F^{(l)}_{1,k}$ and $F^{(l)}_{2,k}$ such that

$$F^{(l)}_{1,k} \approx H^{(l)}_{2,k} \cdot U^{(l)}_1 \cdot \lambda^{(l)}_k$$

(11)

$$F^{(l)}_{2,k} \approx X^{(l)}_k \cdot H^{(l)}_{2,k} \cdot U^{(l)}_2 \cdot \lambda^{(l)}_k^{-1},$$

(12)

where $\lambda^{(l)}_k \in C$. Due to the presence of the noise (10) is only approximately a Kronecker product. The factor matrices are just estimates.

Then the channels $H^{(l)}_{2,k}$ and $H^{(l)}_{2,k}$ can be uniquely estimated (up to a scaling ambiguity) if $U^{(l)}_1$, $U^{(l)}_2$, and $X_k$ are square or flat matrices, respectively. That means $I_1 \geq M_k$, $I_2 \geq M_k$, and $N_R \geq M_k$; Moreover, if $U^{(l)}_1$, $U^{(l)}_2$, and $X_k$ have orthogonal rows, we can avoid the computation of a matrix inverse. Since it is required that $L_1 I_1 \geq N_R \geq L_1 I_2$, to reduce

The number of required training phases we should choose $I_1$ and $I_2$ as small as possible. Therefore, we choose $I_1 = I_2 = M_k$ and $L_1 I_3 = N_R = L_1 M_k$.

Inspired by the tensor-based channel estimation (TENCE) scheme in [6], we show that a proper design of $U^{(l)}_1$ and $U^{(l)}_2$ can reduce the scalar ambiguity to only a sign ambiguity. To do so, we first multiply both sides of (12) by $X^{(l)}_k$ and obtain

$$X^{(l)}_k \cdot F^{(l)}_{2,k} = H^{(l)}_{2,k} \cdot U^{(l)}_1 \cdot \lambda^{(l)}_k^{-1}.$$  

(14)

Let $X^{(l)}_k \in C^{M_k \times I_2}$ contains the first $M_k$ rows of $(X^{(l)}_k \cdot F^{(l)}_{2,k})$. From (14) we have $F^{(l)}_{2,k} = H^{(l)}_{2,k} \cdot X^{(l)}_k \cdot F^{(l)}_{2,k}$. Now we can compute $H^{(l)}_{2,k}$ as

$$\lambda^{(l)}_k \cdot F^{(l)}_{2,k} \cdot U^{(l)}_2 \approx H^{(l)}_{2,k}.$$  

(15)

Inserting (15) into (11), we obtain

$$F^{(l)}_{1,k} \approx \lambda^{(l)}_k \cdot Y^{(l)}_k \cdot F^{(l)}_{2,k} \cdot U^{(l)}_2 + U^{(l)}_1.$$  

(16)

Equation (16) implies that we need to estimate one scalar $\lambda^{(l)}_k$ from a system of equations. If $U^{(l)}_1$ is an orthogonal matrix, we can select $U^{(l)}_1 = U^{(l)}_1$ such that $U^{(l)}_1 \cdot U^{(l)}_1$ is a scaled identity matrix. In the presence of noise in the system, equation (16) holds only approximately since both $F^{(l)}_{1,k}$ and $F^{(l)}_{2,k}$ are noise corrupted. To find $\lambda^{(l)}_k$, we propose to apply a least squares (LS) method. To this end, we first reformulate (16) as

$$\vec{\{F^{(l)}_1\}} \approx \lambda^{(l)}_k \cdot \vec{\{F^{(l)}_2\}} \cdot U^{(l)}_2 + \vec{\{U^{(l)}_1\}}.$$  

(17)

Then $\lambda^{(l)}_k$ is calculated as $\lambda^{(l)}_k = \left(\vec{\{F^{(l)}_2 \cdot U^{(l)}_2 \}}^\top \cdot \vec{\{F^{(l)}_1\}}\right)^{-1} \cdot \vec{\{F^{(l)}_1\}}$. Finally, $\lambda^{(l)}_k$ is computed as the complex square root of $\lambda^{(l)}_k^2$. Note that the complex square root is not unique. Thus, this results in a sign ambiguity.

Algorithm 1 Least squares factorization of a Kronecker product

1. Consider a matrix $C \in C^{M \times N \times P \times Q}$ which is an approximation of the Kronecker product between a matrix $A \in C^{M \times P}$ and $B \in C^{N \times Q}$, i.e., $C \approx A \otimes B$.

2. **Main step**:

3. Define a matrix $\hat{C} = \left[D^2_{1} \ldots D^2_{P}\right]^T \in C^{M \times P \times Q}$. The matrix $D_p \in C^{M \times N \times Q}$ ($p \in \{1, \ldots, P\}$) is computed as $D_p = \left[\text{vec}\{C_{1,p}\} \ldots \text{vec}\{C_{M,p}\}\right]^T$. The matrix $C_{m,p}$ ($m \in \{1, \ldots, M\}$) is the $(m,p)$-th sub-matrix of $C$ and we have

$$C = \left[\begin{array}{ccc} C_{1,1} & C_{1,2} & \cdots & C_{1,P} \\ \vdots & \vdots & \ddots & \vdots \\ C_{M,1} & C_{M,2} & \cdots & C_{M,P} \end{array}\right].$$  

(13)

Then it can be derived that $\hat{C} = \text{vec}\{A\} \cdot \text{vec}\{B\}^T$.

4. Compute the SVD as $C = U \Sigma V^H$. Then the best rank-one approximation of $\hat{C}$ by truncating the SVD, i.e., $\hat{\bar{C}} = u_1 \cdot \sqrt{\sigma_1} \cdot v_1^H$, where $u_1$ and $v_1$ are the first columns of $U$ and $V$, respectively, and $\sigma_1$ is the largest singular value.

5. Finally, we obtain $A = \text{unvec}_{M \times P}\{\hat{\bar{C}}\}$ and $B = \text{unvec}_{N \times Q}\{\hat{\bar{C}}\}$.
III-B. CP decomposition based solution

When \( \hat{U} \) does not have a full column rank, it is still possible to isolate the partition-wise Kronecker product from (7). This fact holds as long as \( [Y]_{\ell \ell} \) lies in the column space of \( \hat{U} \) and the noise is not present. It is worth mentioning that it is possible to reduce the required \( N_{\text{tr}} \) in such a case. We propose a special choice of \( S^{(\ell)} \), which guarantees that the Kronecker product can be isolated while \( \hat{U} \) has a low rank. This idea is based on the CP decomposition.

Define \( I_{\ell \ell} = I_{\ell \ell} = I_{\ell \ell} = r_{\ell} \). Let \( \mathbf{X}^{(\ell)} \in \mathbb{C}^{G_{\ell \ell} \times r_{\ell} \times r_{\ell}} \) be an identity tensor for \( 1 \leq \ell \leq L \). Then the Tucker decomposition in (5) becomes a CP decomposition as defined in [6]. The received signal model (6) is rewritten as

\[
\mathbf{Y}_{k} \approx \sum_{\ell=1}^{L} \mathbf{X}^{(\ell)} \mathbf{1}_{1, k} \mathbf{2}_{2, k} \mathbf{3}_{\ell},
\]

where

\[
\mathbf{X}^{(\ell)} \approx \mathbf{U}_{\ell} \mathbf{U}_{\ell}^{\dagger} \mathbf{Z}_{\ell}.
\]

Equation (18) shows that by applying the CP decomposition to each tensor \( \mathbf{G}^{(\ell)} \), the equivalent model corresponds to a new CP decomposition. Then the TENCE scheme in [6], which is used to estimate the channels for the TWR scheme with just one relay, can be extended to our scenario. In the following we briefly present the estimation procedure.

According to [6], the 3-mode unfolding of \( \mathbf{Y}_{k} \) in (18) can be written as

\[
[\mathbf{Y}]_{3, k} \approx [\bar{U}_{1, k} \mathbf{U}_{2, k}]^{T},
\]

where \( \bar{U}_{1, k} \mathbf{U}_{2, k} \) are flat or square matrices. This requires that \( N_{\ell} \) is set to \( M_{R} \times M_{R} \), \( M_{R} \times M_{R} \), and \( M_{R}^{2} \times M_{R}^{2} \) DFT matrices, \( \forall \ell \). The matrix \( \mathbf{U}_{3, k} \) consists of \( M_{R}^{2} \) columns, that is, the \( \ell \)-th sub-matrix of the identity matrix \( I_{M_{R}^{2}} \).

When the CP-based method is used, the matrix \( \mathbf{U}_{3, k}^{(f)} \) is set to \( \mathbf{U}_{3, k}^{(f)} = I_{M_{R}^{2}} \). The matrix \( \mathbf{U}_{3, k}^{(f)} \) is set to a \( L \) \( M_{R} \times M_{R} \) DFT matrix, where \( \mathbf{U}_{3, k}^{(f)} \) is its \( \ell \)-th sub-matrix that consists of \( M_{R}^{2} \) columns. The matrix \( \mathbf{U}_{3, k}^{(f)} \) is computed in the same way as \( \mathbf{G}_{\ell} \) in [6] and thus we will not repeat it here.

As a measure of the accuracy, we compute the relative estimation error (RSE) defined as

\[
e_{\mathbf{H}, \hat{\mathbf{H}}} = \min_{p \in \{1, -1\}} \frac{\| \mathbf{H} - p\hat{\mathbf{H}} \|_{F}}{\| \mathbf{H} \|_{F}},
\]

and

\[
\mathbf{G}_{k} = \mathbf{G}_{k}^{(\text{com})} \times \mathbf{H}_{k}^{(\text{com})} \times \mathbf{H}_{k}^{(\text{com})T} + \mathcal{N}_{k}
\]

where \( \mathbf{G}^{(\text{com})} = [\mathbf{G}^{(1)} \ldots \mathcal{O} \ldots \mathcal{O} \ldots \mathcal{O} \ldots \mathcal{O}] \in \mathbb{C}^{LM_{R} \times LM_{R} \times N_{\text{tr}}} \), \( \mathbf{H}_{k}^{(\text{com})} = [\mathbf{H}_{k}^{(1)T} \ldots \mathbf{H}_{k}^{(L)T}] \in \mathbb{C}^{M_{R} \times LM_{R}} \), and \( \mathbf{H}_{k}^{(\text{com})T} \) are the conjugate transpose of \( \mathbf{H}_{k}^{(\text{com})} \).

The SLS refinement does not depend on the decompositions of the relay amplification tensors. Finally, all the simulation results are averaged over 10,000 channel realizations.

Figures 2 and 3 demonstrate the accuracy of the proposed BioCE framework under different settings, where \( \mathbf{H}^{(\text{Avg})} \) represents an average over all \( \mathbf{H}_{k}^{(f)}, \forall \ell \). With the given tensor construction, it can be observed that the Tucker based solution provides better channel estimation accuracy than the CP based solution even if they use the same number of training frames \( N_{\text{tr}} \). However, after applying the SLS based enhancements, the CP based solution has gained more than the Tucker based solution.

V. CONCLUSION

In this paper, we have developed a BioCE framework, which consists of the Tucker decomposition based solution and the CP decomposition based solution, to estimate the channels in a two-way non-regenerative network with multiple relays. The BioCE framework provides algebraic estimation methods as well as design rules of the relay amplification tensors. All ambiguities can be resolved up to one sign ambiguity for each relay regardless.
Numerical results show that using the proposed design of the relay amplification tensors the Tucker based method outperforms the CP based method. But using the SLS based refinement the CP based solution achieves better performance than the Tucker based solution.

whether a Tucker based method or a CP based method is used. Numerical results show that using the proposed design of the relay amplification tensors the Tucker based method outperforms the CP based method. But using the SLS based refinement the CP based solution achieves better performance than the Tucker based solution.

VI. REFERENCES