FEEDBACK REDUCTION METHODS FOR DISTRIBUTED COOPERATIVE ANTENNA SYSTEMS WITH TEMPORALLY-CORRELATED CHANNELS

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ABSTRACT

We investigate distributed cooperative antenna (COOPA) systems with limited feedback. A feedback link is required to provide channel state information, and in many cases it is based on the codebook which consists of maximally spaced codes. The feedback overhead can be reduced by organizing the codebook structure in a smart way to exploit the temporal correlation of the channel. We have proposed the combined feedback scheme in our previous work. In this paper we suggest a method which works as a useful guideline to determine the best feedback periods. We also propose a recursive codebook design which has a subspace tracking capability. The simulation results show that the tracking capability of the feedback allows the feedback overhead to be reduced by more than 30% compared with the combined codebook.

1. INTRODUCTION

Cooperative antenna (COOPA) systems have been investigated recently as a means to provide high spectral efficiency in cellular downlink systems [1]-[2]. In COOPA systems, several adjacent base stations (BSs) cooperate in order to support multiple mobile stations (MSs) which are located in the corresponding cooperative area (CA). Therefore, COOPA systems can also be regarded as a multi-user multiple-input multiple-output (MU-MIMO) system, in which multiple transmit antennas at the BS, which are conventionally considered to be located in one BS, are spread over several BSs. COOPA systems have advantageous features compared with conventional cellular systems, like increased degrees of freedom, better intercell interference (ICI) cancelation performance, the rank enhancement effect of the channel matrix, etc. [1]. The biggest gain is acquired from the ICI mitigation effect which allows the links to operate in the high signal to interference plus noise ratio (SINR) regime. This enables COOPA systems to enjoy a great improvement in the spectral efficiency.

In this paper we suggest a method which works as a useful guideline to determine the best feedback periods. We also propose a recursive codebook design which has a subspace tracking capability. The simulation results show that the tracking capability of the feedback allows the feedback overhead to be reduced by more than 30% compared with the combined codebook.

In [1], distributed organization methods have been suggested. One of the main challenges of the distributed COOPA system with transmit filtering is downlink channel estimation. All of the associated BSs in the CA need to know the full channel state information to calculate the corresponding precoding weight matrix. This information has to be transferred from MSs to BSs by using uplink resources. As several BSs and several MSs are involved in COOPA systems and each BS and MS may be equipped with multiple antennas, the number of channel state parameters to be fed back is expected to be high. In an effort to reduce the amount of feedback, the subspace-based channel quantization method has been suggested and evaluated in [3], and has been proven to improve the system performance significantly, compared to the analog pilot retransmission method with relatively small feedback overhead. It allows a MS to select a code from the previously defined codebook based on the current channel state information. The proposed methods are applicable not only to the distributed COOPA systems but also to MU-MIMO systems.

A hierarchical codebook design method, which exploits the temporal correlation of the channel, is proposed as a way of reducing the feedback overhead [3]. It constructs the codebook with a hierarchical structure so that the feedback index can be divided into two parts, i.e., a coarse feedback which points to the codebook index and a fine feedback which indicates the code index. The resulting codebook is termed the combined codebook. It has been shown that we can save a significant amount of feedback resources while maintaining the same performance level as in the case with fully loaded feedback. However, the decision of the optimum feedback period for the combined codebook has not been addressed in the previous work. In [4], a framework is proposed for analyzing multiple antenna limited feedback systems with temporally correlated flat fading channels, and it consists of analytical results concerning the feedback rate. The feedback rate formula is, though, not completely analytic but has a variable which should be obtained by empirical methods like Monte Carlo simulations. On the other hand, the combined codebook can be vulnerable to the border effect, which is explained in detail in Section 4. Thus, there is room for further improvement in designing the feedback overhead reduction method which exploits the temporal correlation of the channel.

In [5], two feedback reduction methods are proposed for MIMO-OFDM systems, exploiting the fact that the channel responses across OFDM subcarriers are correlated. One of them is recursive feedback encoding that selects the optimal beamforming/precoding choices sequentially across the subcarriers, and adopts a small-size time-varying codebook per subcarrier depending on prior decisions. This method can be easily modified to our case to build a state dependent codebook over continuous time instances.

In this paper, we propose a feedback period decision method for the combined codebook and a recursive codebook design method which requires a smaller feedback overhead than the hierarchical codebook design method. The proposed schemes can be used for MU-MIMO systems as well, which involve one BS for downlink data transmission.

Notation: Vectors and matrices are denoted by lower case bold and capital bold letters, respectively. \((\cdot)^T\) and \((\cdot)^H\) denote transpose and Hermitian transpose, respectively. \(\| \cdot \|_F\) denotes the Frobenius norm of a matrix. \(\text{sign}(\cdot)\) stands for the \((i,j)\)th entry of a matrix. \(|S|\) is the cardinality of a set \(S\). \((f(x))^{-1}\) denotes the inverse function of \(f(x)\).

2. SYSTEM MODEL

We consider a precoded MU-MIMO system in which a group of BSs transmits data to multiple MSs simultaneously. Each of \(N_{\text{BS}}\) BSs and each of \(N_{\text{MS}}\) MSs have \(N_{\text{tx}}\) and \(N_{\text{rx}}\) antennas, respectively. The data symbol block, \(s = [s_1, \ldots, s_{N_{\text{TX}}}]^T\) with \(N_{\text{TX}} = N_{\text{MS}} N_{\text{BS}}\), is precoded by a \(N_{\text{TX}} \times N_{\text{rx}}\) matrix \(W\) with \(N_{\text{rx}} = N_{\text{BS}} N_{\text{rx}}\), in case that the number of data streams for each \(n_{\text{tx}}\) \((n_{\text{tx}} \leq N_{\text{ts}})\) is \(N_{\text{rs}}\). Here the first \(N_{\text{rs}}\) data symbols are intended for the first user, the next \(N_{\text{rs}}\) symbols for the second user, and so on. When denoting \(i_{\text{MS}}/i_{\text{BS}}\) as the BS/MS index and \(i_{\text{tx}}/i_{\text{rx}}\) as the transmit/receive antenna index, respectively, we can denote \(i_{\text{tx}}\) as the transmit coefficient between the \(i_{\text{tx}}\)th transmit antenna of the \(i_{\text{MS}}\)th MS...
and the $i$th transmit antenna of the $i$th BS. The $N_s, N_t$ channel coefficients can be expressed as the $N_t \times N_r$ channel matrix $H$ with $[H]_{i,j} = h_{i,j}$. The received signals on $N_r$ receive antennas which are collected in the vector $y$ can be formulated as

$$y = HWs + n,$$

(1)

where $n$ is additive white Gaussian noise (AWGN). The overall channel matrix $H$ is a $N_{BS}N_r \times N_{BS}N_s$ matrix, and is composed of the channel matrices for each user, which are of size $N_t \times N_{BS}N_s$. Equation (2) depicts this relationship.

$$H = [H_1, \ldots, H_j, \ldots, H_{N_{BS}}]^T, \quad j: \text{user index}$$

(2)

Here $H_j$ is the transpose of the channel matrix for user $j$, which is a $N_{BS}N_r \times N_r$ matrix.

The system model is depicted in Fig. 1. The $N_{BS}$ BSs need overall downlink channel state information $H$ for the calculation of the precoding matrix $W$ to form multiple spatial beams which enable independent and decoupled data streams for $N_{MS}$ users. The individual user $j$ estimates the channel $H_j$ and quantizes it by finding the best candidate from the predefined set of codes $C_i$. The index of the chosen code $i_j$ is sent back to the BSs through the limited feedback channel. The BSs reconstruct the channel matrix $H_j$ by looking up the codebook, which is shared by transmitters and receivers. This reconstructed channel matrix is used for the calculation of the precoding matrix $W$. The channel quantization process at MS $j$ can be formulated as

$$H_j = \hat{Q}(U_j^{(S)}) = \arg \min_{C_i \in C_j} d_C(U_j^{(S)}, C_i)$$

(3)

where $C$ is the codebook of size $N (N = 2^{\text{bit}})$ which has the code $C_i \in C_{N_r \times N_{BS}}$ as its elements, and $d_C(\cdot, \cdot)$ denotes the chordal distance which is defined as

$$d_C(T_i, T_j) = \frac{1}{\sqrt{2}} ||T_i T_j^H - T_j T_j^H||_F$$

(4)

for matrices $T_i, T_j$ which have orthonormal columns. $U_j^{(S)}$ is the basis vectors spanning the column space of $H_j$ ($H_j : N_r \times N_{BS}$, $U_j : N_r \times N_r$, $U_j^{(S)} : N_r \times N_{BS}$), and it can be acquired via a singular value decomposition (SVD) of $H_j$ as shown in (5). Here, the superscripts $(S)$ and $(O)$ are used to denote a basis for the signal subspace and the nullspace, respectively.

$$H_j = U_j \Sigma_j V_j^H, \quad \text{where} \quad U_j = [ \ U_j^{(S)} \ U_j^{(O)} ]$$

(5)

3. HIERARCHICAL CODEBOOK DESIGN METHOD

3.1 Combined Codebook Design

For the convenience of the reader, we briefly explain a hierarchical codebook design method [3], which exploits the temporal correlation of the channel as a way of reducing the feedback overhead. It entails a two-layered feedback scheme, e.g., the coarse feedback and the fine feedback. The overall codebook consists of the coarse codebooks, each of which is a set of fine codes. The coarse feedback and the fine feedback. The overall codebook consists of the coarse encoding region is the same for all candidate encoding regions. The $n_t = n_c + n_f$ bit combined codebook, as a whole, is designed to be composed of $2^{n_c}$ groups of the fine codes, and each fine codebook group consists of $2^{n_f}$ fine codes. The feedback operation works as follows. For every coarse feedback period $\tau_c$ of the $n_c$ bit coarse codebook, the MS sends the coarse codebook index $i_c (n_c \text{ bit})$ back to the BSs to indicate the fine codebook group index to which the subsequent fine code indices belong. For every fine feedback period $\tau_f$, the MS sends the fine code index $i_f (n_f \text{ bit})$ to indicate the fine code index of the chosen fine codebook group. Since that $n_c \text{ bit feedback is sent back for every } \tau_c$, and only $n_f \text{ bit extra feedback is needed for every } \tau_f$, we can save uplink resources when compared to the case of sending back $n_c \text{ bit feedback at every time.}$

3.2 Feedback Period Decision Method

This section investigates how to determine the optimum feedback periods $\tau_c$ and $\tau_f$ to guarantee the target performance for a given profile, e.g., the dimension of the downlink channel to be quantized $H_j \in C_{N_r \times N_{BS}}$, the carrier frequency $f_c$, the speed of the MS $v$, and the number of feedback bits $n_c$ and $n_f$. The proposed method is a semi-analytic optimization scheme which is based on the complementary envelope autocorrelation coefficient (CEAC) function, and it works as follows:

First, find the reference target performance-achieving feedback period $\tau_c$ for given $n_c$ and $v$, by simulations. Let us assume we have $\tau_c = 10 T_c$ for $n_c = 3$ bits and $v = 10 \text{ m/s}$. Simulation results indicate that we can achieve less than 0.5 dB degradation in terms of 50% outage SINR compared with the $\tau_c = T_c$ case, when applying $\tau_c = 10 T_c$ ($T_c$ is equal to 71.37 $\mu$s, the OFDM symbol duration for 3GPP Long Term Evolution) for a $n_c = 3$ bit codebook with the $v = 10 \text{ m/s}$. This is a tolerable loss, which satisfies our target performance criteria. The MS quantizes the downlink channel $H_j \in C_{2 \times 1}$, which is generated by the spatial channel model (SCM) in the Urban Macro scenario [6].

Second, based on the reference feedback period $\tau_c$ found empirically in step 1, find the feedback period of our interest for given $n_f$ and $v$ by using the CEAC function. The CEAC function, which is denoted as $p(\Delta t)$, is defined as follows:

$$p(\Delta t) := p(0) - p(\Delta t), \quad \text{for } 0 \leq \Delta t \leq \tau_0,$$

(6)

where $p(\Delta t) = J_0^2 (2\pi v \Delta t / \lambda)$ is the envelope autocorrelation coefficient (EAC) function [7] and $\tau_0$ is the first zero crossing point of $\rho(\Delta t)$ which satisfies $\tau_0 \in \{ \tau_0 : \rho(\tau_0) = 0, \tau_0 \leq \tau_f, \forall \tau < \tau_f, \tau = \}$.
0, 1, 2, · · · , \(\rho(\Delta t)\) represents the correlation between the channel’s responses to sinusoids received with a time separation equal to \(\Delta t\), where \(v\) is the mobile velocity, \(\lambda\) is the wavelength of the propagating signal, and \(J_0\) is the zero-order Bessel function of first kind\(^1\). Fig. 2 depicts the envelope autocorrelation coefficient as a function of the relative time delay \(\Delta t\). The decrease of the correlation for a longer time separation is clearly identified in this figure. We can regard the CEAC as a measure of how uncorrelated the received signals are, for a time separation \(\Delta t\). The CEAC function \(\rho(\Delta t)\) is a monotonically increasing function from 0 (completely correlated) to 1 (completely uncorrelated) in the interval \([0, \tau_0]\).

We should elaborate the time delay \(\Delta t\) as a function of the number of feedback bits \(n\) (\(n_0\) or \(n_f\)) to take the channel quantizer resolution into account. The time delay is meaningful only when it leads to a channel quantizer output transition, since a feedback message is not necessary when the output does not change over time. Hence we define \(\Delta t(n)\) as a minimum recognizable time delay, which is the average of minimum time delays such that the output of the \(n\) bit channel quantizer changes. By inserting \(\Delta t(n)\) into (6), we can acquire \(\rho(\Delta t(n))\) which is a measure of uncorrelatedness corresponding to the minimum recognizable time delay for a \(n\) bit channel quantizer.

It is not a trivial task to come up with a completely analytic formula \(\rho(\Delta t(n))\) in terms of \(n\). However, the relative formula can be drawn as follows. Our empirical research shows that \(\rho(\Delta t(n))\) is inversely proportional to the resolution \(2^n\) of the \(n\) bit channel quantizer.

Once we have a reference case (\(\Delta t(n')\) for \(n'\)) which is acquired by the empirical approach, we can find \(\Delta t(n)\) for a given \(n\) by exploiting (7) as follows. Let us assume that we have a reference recognizable time delay \(\Delta t(n')\) which is empirically proven to achieve the target performance, e.g., less than 0.5 dB degradation from the ideal case, for the given number of feedback bits \(n'\). The CEAC value \(\rho(\Delta t(n))\) for \(n\) bit feedback can be calculated as the following:

\[
\rho(\Delta t(n)) = \frac{\rho(\Delta t(n'))}{2^{n-n'}}
\]  

This means that the level of uncorrelatedness for \(n\) feedback bits should be reduced to 1 over \(2^n-n'\) of that for \(n'\) feedback bits. In other words, the level of correlation should be increased \(2^{n-n'}\) times, since one encoding region is divided into \(2^n-n'\) encoding regions, and this increase of the channel quantizer resolution should be reflected in \(p\). The linear relationship in (8) comes from (7).

Equation (8) allows us to calculate the CEAC value, once the reference value is given. Now we have a \(p\) value of our interest \(n\), so we can find a \(\rho(\Delta t(\lambda)/\lambda)\) value by using the inverse function of (6) for \(0 \leq \Delta t \leq \tau_0\). The whole process can be summarized as follows:

1. Calculate the CEAC value for the target performance-achieving reference case.

\[
\rho(\Delta t(n')) = 1 - \rho(\Delta t(n')) = 1 - J_0^2 \left( \frac{2v\Delta t(n')}{\lambda} \right)
\]  

2. Calculate the CEAC value for the case of our interest by (8).

3. Calculate the normalized delay value \(v\Delta t(n)/\lambda\) to get the feedback period of our interest by using the inverse CEAC function.

\[
v\Delta t(n)/\lambda = \frac{1}{2\pi} \left( J_0^2 (1 - \rho(\Delta t(n))) \right)^{-1}, \text{ for } 0 \leq p \leq 1
\]  

Equation (10) is depicted in Fig. 3. We should note that we can acquire \(\Delta t(n)\) for a given \(v\) by (10).

\(^1\)The derivation of \(\rho(\Delta t)\) is based on the assumption of a uniform scattering environment without a dominant LOS component. The proposed method, hence, is applicable to the uncorrelated Rayleigh fading case.

\(^2\)A channel profile is defined as a channel related quantization result, which can be a precoding matrix or a channel matrix.

4. Recursive Codebook Design Method

The recursive codebook design method is proposed in this section. We introduce a recursive feedback encoding scheme which provides the codebook with the subspace tracking capability. This feature leads to a further reduction of the feedback overhead compared with the combined codebook design method. The recursive method induces a flexibility into the formation of the codebook to address the border effect problem of the combined codebook. This border effect refers to the phenomenon that the possibility that the next channel quantization (CQ) result falls into a different coarse encoding region is higher in the case that the current CQ result points to the encoding region located near the border between different coarse encoding regions than in the case when it points to a code located in the interior of the encoding region.

4.1 Analytical Framework

The downlink temporally correlated channel can be modeled using a Markov chain [4]. The important definitions which will appear in the process of our analysis are as follows:

- \(H_p\) downlink CSI at time \(p\).
- \(J_p\) channel state at time \(p\), \(J_p \in \mathbb{J}\), \(\mathbb{J}\) channel space.
- \(C\) codebook containing \(N\) channel profiles.
- \(C_0\) \(i\)th member of the codebook, \(i \in \{1, \cdots, N\}\).
- \(C'\) recursive codebook, \(C' \subset C\)
where $N_r$ is the size of the codebook which corresponds to the number of channel states.

### 4.2 Recursive Codebook Design

The proposed recursive codebook design method does not exploit any channel statistical information. It takes an existing codebook $\mathcal{C}$ of size $N_t$ and reorganize it such that there exists a small size codebook which is decided by the current state for each codeword. This small size adaptive codebook $\mathcal{C}^r$, termed recursive codebook, is a subset of the whole codebook $\mathcal{C}$. The construction/operation procedure of the recursive codebook is as follows.

I. (Codebook Construction Phase) We perform the codebook construction algorithm for a given training sequence $T$ to get $N_t = 2^m$ codewords. In our case, we use the modified Linde, Buzo, and Gray (mLBG) algorithm for the codebook generation. The mLBG algorithm takes $T$ and $N_t$ as input parameters, and provides a codebook $\mathcal{C}$ as an output [3]. Thus, this can be formulated as follows.

$$(\mathcal{C}) = \text{mLBG}(T, N_t) \quad (11)$$

II. (Subspace Adjusting Mode) Set the time index $p = 0$. The initial quantization is performed with the full size codebook $\mathcal{C}$, which requires $n_t$ feedback bits. The quantization procedure with $\mathcal{C}$ can be rewritten as

$$C^o_{p0} = \mathcal{Q}(H_p, C) = \arg \min_{C_i \in \mathcal{C}} d_r(U_p^{(s)}, C_i). \quad (12)$$

In this case we adopt the subspace based channel quantization method as an example. This step is repeated for every $\tau_a$ (subspace adjusting feedback period) in an effort to prevent the codebook from losing track of the channel state.

III. (Subspace Tracking Mode)

1. We construct a recursive codebook $\mathcal{C}^r[\mathcal{J}_p]$ for the given Markov state $\mathcal{J}_p$ at time $p$ as a subset of $\mathcal{C}$ as follows:

$$C^r[\mathcal{J}_p] = \{ C_{i | \forall i \in N_r^j(\epsilon) \text{ for } j = \mathcal{J}_p} \} \quad (13)$$

Here, the next step $\epsilon$-neighborhood of the Markov state $j$, $N_r^j(\epsilon)$, is defined as

$$N_r^j(\epsilon) = \{ i \in J | d_r(C_i, C_j) \leq \epsilon \}, \quad j \in \{1, \ldots, N_r\}, \quad (14)$$

where $\epsilon$ is taken such that the condition $|N_r^j(\epsilon)| = N_r$ is satisfied. $N_r^j(\epsilon)$ is the set of Markov states at time $p$ for the given Markov state $j$ at time $p - 1$. $N_r = 2^{\tau_a}$ is the size of the recursive codebook, which has a fixed value for implementational convenience. Therefore, $C^r[\mathcal{J}_p]$ is a collection of $N_r$ codewords chosen from $\mathcal{C}$ which are closest to $C_{\mathcal{J}_p}$. In this case, we have chosen the chordal distance $d_r(T, T_j)$ as the distance measure.

We should note that the recursive codebook $C^r[\mathcal{J}_p]$ is centered around $C_{\mathcal{J}_p}$ and includes itself. The recursive codebook is dependent on the current state $\mathcal{J}_p$ and is of the same size irrespective of $\mathcal{J}_p$, and it has a tracking capability (state-dependent codebook), which is illustrated in Fig. 4.

2. Subspace tracking feedback period $\tau_a$ elapses. Set $p = p + 1$.

3. The quantization procedure is performed with the recursive codebook $C^r[\mathcal{J}_{p-1}]$ which is acquired by step 1. We should note that it requires only $n_t$ feedback bits instead of $n_t$ bits.

$$C^o_{p1} = \mathcal{Q}(H_p, C^r[\mathcal{J}_{p-1}]) = \arg \min_{C_i \in C^r[\mathcal{J}_{p-1}]} d_r(U_p^{(s)}, C_i) \quad (15)$$

4. If the actual time $t[p]$ at the time index $p$ reaches a subspace adjusting feedback period $\tau_a$, go back to step II (subspace adjusting mode). Fig. 5 depicts the corresponding time frame. Otherwise, set the state transition as (16) and go back to step 1 (subspace tracking mode), which allows the recursive codebook $C^r[\mathcal{J}_p]$ at the next time index to be centered around the most recent codeword $C^o_{p1}$.

$$J_p = j, \quad \text{if } C^o_{p1} = C_j \quad (16)$$
The performance of the combined codebook is shown in Fig. 6. The combined codebooks are provided by the modified LBG VQ algorithm and the hierarchical codebook construction method as described in [3]. The combined codebook of the \( n_c \) bit coarse and \( n_f \) bit fine feedback with corresponding feedback periods \( \tau_c \) and \( \tau_f \) is denoted by \( n_c + n_f \) bCQ \( ([\tau_c], [\tau_f]) \), where the unit of the feedback period is the number of OFDM symbols: \( [\tau] = m \) when \( \tau = m T_s \). The feedback periods are determined by the feedback period decision method in Section 3. At 50 \% outage SINR, the 5 bit codebook case (5bCQ), which is generated by the hierarchical codebook construction method, shows a 3.9 dB gain over the centralized CA case (cCA). In this case, the 5 bit feedback is sent back for every symbol. The 3+2 bit combined codebook with the optimum feedback period pair \( ([\tau_c], [\tau_f]) \in \{ (10,5), (10,10), (20,5) \} \) is less than 0.1 dB away from the performance of the 5bCQ case. In terms of the required resources, the 5bCQ case requires 5 bits/symbol for feedback, while the 3+2bCQ case needs only 0.7 bits/symbol. Thus, the 3+2 bit combined codebook can achieve the performance of the 5 bit codebook with a negligible degradation by using just 14 \% of the feedback resources. We have also tested the 3+2bCQ case with various suboptimum feedback periods, e.g., \( (10,10) \) and \( (20,5) \), and each case is denoted by the subscript 2 and 3, respectively. None of these cases outperforms the optimum case (10,5), and the performance degradations are 0.7 dB and 0.8 dB for the longer fine feedback period case and the longer coarse feedback period case, respectively. We should note that the optimum feedback period pair guarantees the target performance. In this case, \( ([\tau_c], [\tau_f]) \in \{ (10,5) \} \) is the optimum feedback period pair, and any other case with a longer period results in a performance degradation.

The performance of the recursive codebook is shown in Fig. 7. The recursive codebook of the \( n_t \) bit subspace adjusting and \( n_c \) bit subspace tracking feedback with corresponding feedback periods \( \tau_a \) and \( \tau_f \) is denoted by \( n_t + n_c \) bCQ \( ([\tau_a], [\tau_f]) \). Simulation results show that thanks to the channel tracking capability the recursive codebook 5-3bCQ (150,5) achieves the same performance as that of the combined codebook 3+2bCQ (10,5) case with smaller feedback overhead \( ((5 + 3 \cdot (2^{22} - 1))/150 = 0.61 \text{ bits/symbol}) \). The feedback overhead reduction ratio in comparison with the combined codebook of the same performance is 12.4 \%. If a more relaxed target is allowed (50 \% outage SINR: 18.4 dB), the feedback overhead saving ratio is 36 \%. The recursive codebook, in short, outperforms the combined codebook in terms of the required feedback resources.

6. CONCLUSION

We have introduced a feedback period decision method for the combined codebook which is very effective in finding an optimum feedback period for the given target performance achieving reference point. We have also proposed a recursive codebook design method which addresses a border effect problem of the combined codebook. The recursive codebook requires much less feedback overhead compared with the combined codebook, due to its tracking capability.

REFERENCES