A Channel Matching Based Hybrid Analog-Digital Strategy for Massive Multi-User MIMO Downlink Systems

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Abstract—in this paper we study the downlink of a hybrid analog-digital massive multi-user MIMO (MU-MIMO) system. An efficient hybrid strategy is proposed, where the analog beamforming matrix is determined using a channel matching criterion while the digital beamforming consists of pre-filters and post-filters. The digital post-filters are executed using algorithms and the digital matrices are constrained to have a low complexity codebook based hybrid precoding scheme is introduced for the mmWave multi-user MIMO (MU-MIMO) downlink channel. In [10] a low complexity hybrid transmit strategy is proposed for the same scenario. However, only single antenna receivers and the zero forcing (ZF) transmit strategy are considered in [10] and [9]. Hence, this motivates us to develop a more general hybrid analog-digital strategy for the massive MU-MIMO downlink channel.

In this paper we develop hybrid precoders and decoders for such a massive MU-MIMO downlink channel to achieve a higher spatial multiplexing gain. The RF precoding is implemented using only phase shifters, i.e., only constant modulus entries are allowed. Consequently, we get the same phase shifts for all users in the equivalent baseband domain. To circumvent these non-convex constraints, we design the analog and digital beamforming matrices sequentially. First, analog precoders and decoders are designed based on a per-user channel matching criterion. Then digital pre-filters and post-filters are derived based on four alternative linear transmit strategies for the MU-MIMO downlink channel followed by an optimal power allocation via the water-filling algorithm. The applied linear transmit strategies include the zero-forcing (ZF) method [11], the MMSE method [12], [13], the block diagonalization (BD) technique [14] and the regularized BD (RBD) technique [15]. Sufficient conditions under which the unconstrained digital solutions can be realized using hybrid precoding and decoding are derived. Simulation results show that the proposed hybrid transmit and receive strategy achieves a good performance compared to its pure digital counterpart.

Notation: The expectation, rank, conjugate, and Hermitian transpose are denoted by $E\{\cdot\}$, $\text{rank}(\cdot)$, $\{\cdot\}^*$, and $\{\cdot\}^H$, respectively. The Euclidean norm of a vector and the Frobenius norm are denoted by $\|\cdot\|_2$ and $\|\cdot\|_F$, respectively. A diagonal matrix and a block diagonal matrix are created by $\text{diag}(\{e\})$ and $\text{blkdiag}(\{A_n\})^{N}$, respectively. The operator $\angle(A)$ computes the angle of $A$ element-wise.

I. INTRODUCTION

Massive MIMO provides significant MIMO gains while maintaining a high energy efficiency [1]. When combined with millimeter wave (mmWave) technology, it will additionally benefit from large chunks of underutilized spectrum in the mmWave band [2]. Hence, mmWave massive MIMO communication is a potential technique for future wireless networks [3]. Assume that the number of deployed RF chains is significantly smaller than the number of antenna elements. To fully exploit the MIMO gain, hybrid analog precoding schemes, realized using phase shifters or switches in the RF domain [4], and digital precoding schemes, implemented in the digital baseband domain as in conventional MIMO, provide a promising performance. Analog circuits using only phase shifters impose constant modulus constraints on the analog beamforming matrices and thus create new challenges for the required signal processing [5], [6]. Several papers have recently studied such hybrid precoding design problems, including [7], [8], [9], [10]. In [7] a compressed sensing based hybrid precoding is proposed to approximate the optimal unconstrained solution of a point-to-point MIMO system and is further refined in [8]. In [9] a low complexity codebook based hybrid precoding scheme is introduced for the mmWave multi-user MIMO (MU-MIMO) downlink channel. In [10] a low complexity hybrid transmit strategy is proposed for the same scenario. However, only single antenna receivers and the zero forcing (ZF) transmit strategy are considered in [10] and [9]. Hence, this motivates us to develop a more general hybrid analog-digital strategy for the massive MU-MIMO downlink channel.
\( R_{a,k} = W_k^H H_k F_k \cdot (W_k^H H_k F_k)^H : \sum_{k=1}^{K} \| F_k \|^2_F \leq P_f. \) (2)

Assuming that perfect channel state information (CSI) is available at both the BS and the UE, dirty paper precoding achieves the capacity of the MU-MIMO downlink channel and thus provides optimal solutions to (2) [16]. However, since it has a high implementation complexity [17], in practice low complexity linear methods are preferred, e.g., ZF [11], MMSE [12], [13], and channel block diagonalizing based methods such as BD [14] and RBD [15]. All these linear algorithms approximately solve (2) by first suppressing the inter-user interference. Thereby, the system will be decoupled into \( K \) parallel single-user MIMO sub-systems. Then each UE designs its precoders and decoders assuming that the residual interference can be treated as noise.

Armed with this understanding, we now turn to the more challenging hybrid massive MIMO formulation. In contrast to the classical model, this formulation assumes that the precoding and decoding are achieved by hybrid analog and digital schemes. The number of RF chains is assumed to be much smaller than the number of antenna elements, i.e., \( M_T \gg N_T \geq N_s \) and \( M_K \gg N_K \geq N_s \) [7]. A baseband precoding matrix \( F_{A,k} \in C^{N_T \times N_s} \) is applied on a per-user basis. Afterwards, the precoded digital signal is upconverted into an analog signal followed by analog circuits, where an RF precoder \( F_A \in C^{N_T \times N_T} \) is implemented using analog phase shifters. Hence, constant modulus constraints should be fulfilled for each element of \( F_A \), i.e., \( |(F_A)_{a,b}| = 1 \) for all \( a \in \{1, \ldots, M_T\} \) and \( b \in \{1, \ldots, N_T\} \). Similarly, hybrid combiners are used at each UE, where the digital combiner is denoted by \( W_k^H \in C^{N_s \times N_K} \) and the corresponding analog combiner is represented by \( W_{A,k}^H \in C^{N_s \times N_s} \). Again, the entries of \( W_{A,k} \) have unit magnitudes. This formulation leads to two additional constraints on (2)

\[
\{ F_A, W_{A,k} \} \in F^{(RF)}
\]
\[
W_k = W_{A,k} F_{B,k}, \quad F_k = F_A F_{B,k},
\] (3a)
\[
W_{B,k} = W_{A,k} \Sigma_{A,k} V_{A,k} \]
\[
F_k = F_A F_{B,k},
\] (3b)

where \( F^{(RF)} \) represents a general set of matrices or vectors with constant modulus entries. Thus, the hybrid precoding and decoding matrices are restricted in two challenging manners. First, the RF matrices have only constant modulus entries. Second, different UEs are coupled via their joint RF solution at the BS. The goal of this paper is to develop efficient linear solutions to problem (2) subject to the non-convex constraints in (3), which is in general NP-hard.

### III. PROPOSED EFFICIENT HYBRID STRATEGY

In this section we introduce the proposed hybrid analog-digital algorithm. First, the RF precoders and decoders are designed based on a channel matching criterion. Then optimal digital precoders and decoders are computed according to design criteria traditionally used for MU-MIMO downlink systems, i.e., BD, RBD, ZF, and MMSE.

Let us define a rank-min\((N_s, N_T)\) truncated SVD of \( H_k \) as

\[
H_k \approx U_{a,k} \Sigma_{a,k} V_{a,k}^H,
\]
where \( U_{a,k} \in C^{M_s \times N_s}, \) \( V_{a,k} \in C^{N_T \times N_s}, \) and \( \Sigma_{a,k} \in R^{N_s \times N_s} \) has non-zero elements only on its main diagonal. Moreover, we have \( \sum_{k=1}^{K} N_{T,k} = N_T. \) Using the channel matching criterion, our proposed design of analog precoders and decoders is then calculated by

\[
W_{A,k} = e^{i\phi(U_{a,k})}, \quad F_{A,k} = e^{i\phi(V_{a,k})},
\] (4)

where \( F_A = [F_{A,1} \quad F_{A,2} \quad \cdots \quad F_{A,K}]. \)

Without loss of generality, the digital precoders and decoders are decomposed into

\[
W_{B,k} = W_{P,k} \cdot W_{D,k}, \quad F_{B,k} = F_P \cdot F_{D,k} \cdot \text{diag}(\sqrt{\gamma_k}),
\]
where \( W_{P,k} \in C^{N_{K} \times N_k} \) and \( F_P \in C^{N_T \times N_T} \) are square pre-filtering matrices. The vector \( \gamma_k = [\gamma_{k,1} \cdots \gamma_{k,N_k}]^T \) represent the power loading onto the eigenmodes of the equivalent channel. Let us calculate the economy-sized SVD of \( W_{A,k} \) as

\[
W_{A,k} = U_{A,k} \Sigma_{A,k} V_{A,k}^H,
\]
where \( U_{A,k} \in C^{M_s \times N_k}, \) \( V_{A,k} \in C^{N_k \times N_k}, \) \( \Sigma_{A,k} \) is a diagonal matrix containing the singular values of \( W_{A,k}, \) and the economy-sized SVD of \( F_A \)

\[
F_A = U_A \Sigma_A V_A^H
\]

where \( U_A \in C^{M_T \times N_T}, \) \( V_A \in C^{N_T \times N_T}, \) \( \Sigma_A \) is a diagonal matrix containing the singular values of \( F_A. \)

Note that rank\((W_{A,k}) = N_k \) and rank\((F_A) = N_T \) are assumed for our definitions. Then we propose

\[
W_{P,k} = V_{A,k} \Sigma_{A,k}^{-1} W_{A,k} \left\| \gamma_k \right\|, \quad F_P = V_A \Sigma_A^{-1}.
\]

Only local CSI is required for the calculation of \( W_{A,k} \) and \( W_{P,k}. \) The proposed pre-filters in (6) provide two advantages. At the receiver side, they whiten the equivalent noise vectors such that

\[
W_{P,k} W_{A,k}^H \in \{z_k z_k^H\} W_{A,k} W_{P,k} = U_{A,k}^H \sigma_k^2 U_{A,k} = \sigma_k^2 I_{N_k}
\]

At the transmitter side, the actual transmit power is not a function of the analog precoder any more, i.e.,

\[
\| F \|^2 = \| F_D \cdot \text{diag}(\sqrt{\gamma}) \|^2,
\]

where \( F_D = [F_{D,1} \quad \cdots \quad F_{D,K}] \in C^{N_T \times N_k} \) and \( \gamma = [\gamma_1^T \cdots \gamma_K^T]^T. \) Post-digital decoders and precoders \( W_{H,k}^D \) and \( F_{D,k} \) are designed for an equivalent MU-MIMO downlink channel, where for the \( k \)-th UE the effective channel is given by

\[
\hat{H}_k = W_{H,k}^H H_k F_k A_k = \frac{U_{A,k}^H H_k F_k A_k}{\sqrt{\gamma_k}}
\]
and the equivalent noise vector is expressed as

\[
\tilde{z}_k = W_{H,k}^H z_k = U_{A,k}^H z_k.
\]

In the following we derive the design criteria dependent digital decoders \( W_{H,k}^D \) and precoders \( F_{D,k}. \)

**BD Based Design:** Taking the \( k \)-th UE as an example, to fulfill the requirement of the BD approach in [14], we decompose \( F_{D,k} \) into

\[
F_{D,k} = F_{D,1,k} \cdot F_{D,2,k},
\]

where \( F_{D,1,k} \) is responsible for inter-user interference avoidance and \( (F_{D,2,k} \cdot \text{diag}(\gamma_k)) \) optimizes the performance of the interference-free sub-system of the \( k \)-th UE.

Let us define the combined effective channel matrix \( \hat{H}_k \) for all UEs except the \( k \)-th UE as

\[
\hat{H}_k = [\hat{H}_k^T \cdots \hat{H}_k^{K,1} \cdots \hat{H}_k^{K,K}]^T \in C^{(N_k-N_k) \times N_T}.
\]

Define rank\((\hat{H}_k) = \tilde{L}_k \) and the SVD of \( \hat{H}_k \) as

\[
\hat{H}_k = \hat{U}_k \hat{\Sigma}_k \hat{V}_k^H = \tilde{U}_k \tilde{\Sigma}_k [\tilde{V}_k^{(1)} \quad \cdots \quad \tilde{V}_k^{(\tilde{L}_k)}] \]

(8)
where \( \hat{V}_k^{(0)} \in \mathbb{C}^{N \times (N_T - L_k)} \) contains the last \((N_T - L_k)\) right singular vectors. The matrix \( \hat{V}_k^{(0)} \) forms an orthogonal basis for the null space of \( \hat{H}_k \). Therefore, we choose
\[
F_{D_1,k} = \hat{V}_k^{(0)}. \tag{9}
\]
Define the SVD of \( \hat{H}_k F_{D_1,k} \) as
\[
\hat{H}_k F_{D_1,k} = U_k \Sigma_k V_k^H. \tag{10}
\]
Then the precoder \( F_{D_2,k} \) and the decoder \( W_{D,k} \) are given by
\[
F_{D_2,k} = \tilde{V}_k, \quad W_{D,k} = \tilde{U}_k, \quad \forall k. \tag{11}
\]
Note that the BD algorithm is applicable only if \( \hat{H}_k \) has a non-empty null space for all \( k \), i.e., \( N_T > \max_k (N_R - N_k) \) if \( \hat{H}_k \) is a full rank matrix.

**RBD Based Design:** Instead of nulling the inter-user interference, the RBD algorithm [15] allows a residual amount of interference in order to balance it with the noise enhancement especially in the low SNR regime.

Similarly as in (7), we decompose \( F_{D,k} \) into \( F_{D,k} = F_{D_1,k} \cdot F_{D_2,k} \). Let us define
\[
F_{D_1} = \begin{bmatrix} F_{D_1,1} & \cdots & F_{D_1,K} \end{bmatrix}, \quad s = \begin{bmatrix} s_1^T & \cdots & s_K^T \end{bmatrix}^T, \quad \text{(12)}
\]
and \( Z = \begin{bmatrix} Z_1^T & \cdots & Z_K^T \end{bmatrix}^T. \) Assume that \( F_{D_2,k} \) is a unitary matrix and equal power allocation is used, e.g., \( \gamma_k = \beta \nu N_k \) for all \( k \), where \( \beta \) is used to fulfill the transmit power constraint. RBD designs \( F_{D_2,k} \) by solving the following optimization problem [15]
\[
\min_{\|F_{D_1,k}\|_F = 1, \forall k} \sum_{k=1}^K \left( \|\hat{H}_k F_{D_1,k}\|_F^2 + \frac{\sigma_n^2 N_R}{P_T} \|F_{D_1,k}\|_F^2 \right). \tag{13}
\]
Using the definition in (8) and inspired by (18), the optimal solution to (13) without scaling is given by
\[
F_{D_1,k} = \tilde{V}_k \left( \Sigma_k^2 \Sigma_k + \frac{\sigma_n^2 N_R}{P_T} I_{N_T} \right)^{-\frac{1}{2}}. \tag{14}
\]
The remaining digital precoders and decoders are designed in the same way as in case of BD. Therefore, the RBD solutions of \( F_{D_2,k} \) and \( W_{D,k} \) can also be represented by (11).

**ZF Based Design:** The ZF criterion eliminates the interference while ignoring the noise [11]. Define
\[
F_{D,k} = \beta \cdot F_{D_1,k}. \tag{15}
\]
Using the definition in (12) and define \( W_{D,k} = \beta^{-1} W_{D_1,k} \), the ZF optimization problem is formulated as
\[
\min_{\beta, F_{D_1,k}, W_{D_1,k}, \forall k} \mathbb{E}\{ \|s - \hat{H}_k F_{D_1,k}s\|_2^2 \}, \quad \text{s.t.} \|F_{D_1,k}\|_F^2 \leq P_T \tag{16}
\]
where
\[
\hat{H}_k = \begin{bmatrix} \hat{H}_{k,1}^T \end{bmatrix} \in \mathbb{C}^{N \times N_T}. \tag{17}
\]
When \( W_{D_1,k} \) is fixed, using the method of Lagrangian multipliers a closed-form solution for \( F_{D_1} \) and \( \beta \) is obtained as
\[
F_{D_1} = \hat{H}_k^H (\hat{H}_k^H \hat{H}_k)^{-1}, \quad \beta = \frac{P_T}{\|F_{D_1}\|_F^2}. \tag{18}
\]
Inserting this optimal solution back into (17), the resulting problem is not a function of \( W_{D_1,k} \) and therefore an arbitrary \( W_{D_1,k} \) is optimal. To avoid coloring the noise, we set \( W_{D_1,k} = [I_{N_k}] \cdot 0 \). Note that the involved channel inversion in (18) requires that \( N_T \geq N_k \).

**MMSE Based Design:** Using the definitions in (15) and (17), the MMSE optimization problem is formulated as
\[
\min_{\beta, F_{D_1,k}, W_{D_1,k}, \forall k} \mathbb{E}\{ \|s - \hat{H}_k F_{D_1,k}s - W_{D,k} z\|_2^2 \}, \quad \text{s.t.} \|F_{D_1,k}\|_F^2 \leq P_T \tag{19}
\]
where \( W_{D} = \text{blkdiag} \{ W_{D,k} \}_{k=1} \). Problem (19) is a non-convex optimization problem [12], [13]. For implementation simplicity, we resort to a suboptimal sequential design. First, the BS computes \( F_{D_1} \) using the MMSE criterion while assuming that \( W_{D_1,k} = \beta^{-1} [I_{N_k}, 0] \). Again, by applying the Lagrangian method a closed-form solution is obtained as
\[
F_{D_1} = \left( \hat{H}_k^H \hat{H}_k + \frac{\sigma_n^2 N_k}{P_T} I_{N_T} \right)^{-1} \hat{H}_k^H, \quad \beta = \sqrt{\frac{P_T}{\|F_{D_1}\|_F^2}}. \tag{20}
\]
Next, for fixed \( F_{D_1,k} \), we compute \( W_{D,k} \) by solving (19). The optimal problem is computed by [12]
\[
W_{D,k} = (\hat{H}_k F_{D_1,k} \hat{H}_k^H)^{-1} \left( \hat{H}_k F_{D_1,k} + \sigma_n^2 I_{N_k} \right), \quad \forall k. \tag{21}
\]
where \( N_{R,k} = \sum_{\nu=1}^{K} \hat{H}_{k} F_{D,\nu,k} \hat{H}_{k}^H + \sigma_n^2 I_{N_k} \). Note that it is difficult for the k-th UE to obtain \( N_{R,k} \) exactly with only local CSI.

**Optimal Power Allocation:** After applying the derived analog and digital beamforming matrices, problem (2) subject to the non-convex constraints in (3) reduces to a power allocation problem. A general formulation of this optimization problem is expressed as
\[
\max_{\gamma_{k,ik}} \sum_{k=1}^{K} \sum_{k=1}^{N_k} \log_2 (1 + \sigma_n^{-2} \sigma_{k,ik}^{-1} \gamma_{k,ik}) \quad \text{s.t.} \sum_{k=1}^{K} \sum_{i=1}^{N_k} \alpha_{k,ik} \gamma_{k,ik} \leq P_T. \tag{22}
\]
The optimal solution of (22) is obtained using the water-filling algorithm [19], i.e., \( \gamma_{k,ik} = \frac{1}{\alpha_{k,ik}} \max (\frac{1}{2} - \frac{\sigma_{k,ik}}{\sigma_{k,ik}, \alpha_{k,ik}}) \) and \( \nu \) is the water level such that \( \sum_{k=1}^{K} \sum_{i=1}^{N_k} \max (\frac{1}{2} - \frac{\sigma_{k,ik}}{\sigma_{k,ik}, \alpha_{k,ik}}) = P_T \).

When BD is used, \( \sigma_{k,ik} = \alpha_{k,ik}^{-1} \) and \( \alpha_{k,ik} = 1, \forall k, i \). When ZF or MMSE is used, we set \( \sigma_{k,ik} = 1, \forall k, i \) and \( \alpha_{k,ik} = \gamma_{k,ik} \) is the \((i_k, i_k)\)-th element of the matrix \( F_{D_1}^H F_{D_1} \). When RBD is used, we set \( \gamma_{k} = \frac{P_T}{\|F_{D_1}\|_F^2} \cdot 1_{N_k} \) for all \( k \).

**IV. UNCONSTRAINED DIGITAL SOLUTIONS AND THEIR EQUIVALENT HYBRID REALIZATIONS FOR SPECIAL CASES**

In this section we discuss optimal unconstrained digital solutions and their exact hybrid analog-digital realizations for some special cases. Let \( F_{\text{full},k} \in \mathbb{C}^{M_T \times M_k} \) and \( W_{\text{full},k} \in \mathbb{C}^{M_k \times M_k} \) represent optimal solutions to problem (2), which are obtained by using BD, RBD, ZF, or MMSE together with equal power allocation. Let \( F_{\text{opt},k} \in \mathbb{C}^{M_T \times N_k} \) and \( W_{\text{opt},k} \in \mathbb{C}^{M_k \times N_k} \) contain the first \( N_k \) columns of \( F_{\text{full},k} \) and \( W_{\text{full},k} \), respectively. Then \( F_{\text{opt},k} = F_{\text{opt},k} \cdot \text{diag}(\sqrt{\nu T}) \) and \( W_{\text{opt},k} \) are our proposed unconstrained digital solutions, where \( \gamma_{k} \) denotes the optimal power allocation. Note that when ZF and MMSE are used, the proposed unconstrained digital solutions are meaningful only if \( \hat{H}_k \) has full rank and \( M_T \geq M_k \).
Inspired by Theorem 1 of [20], we show that optimal hybrid solutions, which achieve the performance of their corresponding unconstrained digital solutions exactly, can be constructed for some special cases.

**Lemma 1:** Let us define $F_{\text{opt}} = [F_{\text{opt},1} \cdots F_{\text{opt},K}] \in \mathbb{C}^{M_T \times N_T}$ and $\text{rank}(F_{\text{opt}}) = r$. An exact matrix decomposition exists such that $F_{\text{opt}} = F_{\text{A, opt}} \cdot F_{\text{B, opt}}$ only if $N_T \geq \frac{2M_TR}{M_T-2r} \cdot r$, where $F_{\text{A, opt}} \in \mathbb{C}^{M_T \times N_T}$ is $\mathbb{F}(r)$. A closed-form solution for this decomposition can be constructed if $N_T \geq 2r$.

**Proof:** Let us define the economy-sized SVD of $F_{\text{opt}}$ as

$$F_{\text{opt}} = U_s \Sigma_s V_s^H,$$

where $U_s \in \mathbb{C}^{M_T \times r}$. Then we rewrite $U_s$ as

$$U_s = \left(\Phi_1 \cdots \Phi_j \cdots \Phi_{\text{res}}\right) \left[\begin{array}{cc} \text{blkdiag}\{\nu_j\}_{j=1}^r & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array}\right],$$

(23)

where $\Phi_j \in \mathbb{C}^{M_T \times 2}$, $\nu_j \in \mathbb{C}^2$, and the elements of $\Phi_{\text{res}} \in \mathbb{C}^{M_T \times (N_T - 2r)}$ have random phases. This formulation requires that the $j$-th column of $U_s$, denoted by $u_{s,j}$, should be factorized as $u_{s,j} = \Phi_j \cdot \nu_j$. A closed-form solution for this decomposition is provided by Theorem 1 in [20]. Lastly, $F_{\text{B, opt}} = \Omega \Sigma_s V_s^H$.

If $F_{\text{A, opt}}$ and $\Omega$ do not have the specific structure as in (23), in general this decomposition holds only if the degrees of freedom of $(F_{\text{A, opt}} \cdot \Omega)$ are not less than the degrees of freedom of $U_s$. Noticing that one complex variable can be treated as two real variables, the number of free real-valued variables in $(F_{\text{A, opt}} \cdot \Omega)$ is $(M_T N_T + 2M_T r - 2r^2)$. The matrix $U_s$ has orthonormal columns, i.e., $u_{s,j}^H u_{s,j} = 0$, $\forall j \neq j, j \in \{1, \cdots, r\}$, and $\|u_{s,j}\|^2 = 1$, $\forall j$. These conditions create $r^2$ constraints thus consume $r^2$ degrees of freedoms. The remaining free variables of $U_s$ are $(2M_T r - 2r^2)$. We should have $2M_T r - 2r^2 \leq M_T N_T + 2M_T r$, which can be reformulated as $N_T \geq \frac{2M_T r - 2r^2}{M_T-2r}$.

This conclusion can be also used for realizing unconstrained digital decoders if $N_k \geq 2r_k$, where $r_k = \text{rank}(W_{\text{opt},k})$.

### V. SIMULATION RESULTS

The proposed algorithms are evaluated using Monte-Carlo simulations. The maximum allowable power $P_T$ is fixed to unity. The SNR is thus defined as $\text{SNR} = 1/\sigma_n^2$. All the simulation results are obtained by averaging over 1000 channel realizations.

The simulated channel is a mmWave channel with a geometric channel model of $L$ = 8 scatterers. Each scatterer contributes to a single propagation path between the BS and the UE. The $k$-th UE's channel is modeled as $H_k = \frac{1}{C} \sum_{\ell=1}^{L} \alpha_{\ell,k} a_{R,\ell,k}(\theta_{R,\ell,k}) a_{T,\ell,k}^H(\theta_{T,\ell,k})$, where $\alpha_{\ell,k}$ is the random complex gain of the $\ell$-th path of the $k$-th UE, with zero-mean and $\mathbb{E}(|a_{R,\ell,k}|^2) = 1$ [8]. The variables $\{\theta_{R,\ell,k}, \theta_{T,\ell,k}\} \in [0, 2\pi)$ denote the angle of arrival and the angle of departure of the $\ell$-th path of the $k$-th UE, respectively. Finally, $\alpha_{\ell,k}$ and $a_{\ell,k}$ are the array steering vectors of the BS and the UEs, respectively. As in [7], a uniform linear array (ULA) geometry is used at both ends. The inter-element spacing of the ULA is equal to half of the wavelength.

Fig. 1 and Fig. 2 compare the performance of the proposed hybrid analog-digital strategy using different linear design criteria. Fig. 1 illustrates that the hybrid solutions have almost the same performance as the corresponding unconstrained digital solutions especially when RBD or BD is used. The performance difference increases as the number of spatial steams increases. Among the four linear strategies, the RBD based design provides the best performance while the ZF based design performs the worst. Moreover, when $N_k = 2$ the settings of $N_T$ and $N_k$ fulfill the requirement of Lemma 1. Then the unconstrained solutions can be achieved exactly using hybrid analog-digital designs. Fig. 2 shows that the gap between the unconstrained digital solutions and the hybrid solutions enlarges when the number of spatial steams increases.

### VI. CONCLUSION

In this paper we have developed a general approach to realize hybrid analog-digital beamforming in a massive MU-MIMO downlink scenario. The proposed approach can be applied regardless of the number of antennas at the BS and at the UE. A multiple-stream transmission can be achieved for each UE. Furthermore, unconstrained digital solutions have been proposed for the same scenario. We have shown that the unconstrained digital solutions have equivalent hybrid analog-digital realizations for some special cases. Simulation results show that the proposed hybrid analog-digital solutions have almost the same performance as their corresponding unconstrained digital counterparts. The MMSE based solution provides a better performance especially in the low SNR regime while the RBD based design achieves a higher data rate especially in the high SNR regime.
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