Frequency Domain Equalization for Dispersive Optical Channels with Intensity Modulation and Direct Detection

Mike Wolf, Sher Ali Cheema, and Martin Haardt

Technische Universität Ilmenau, Communications Research Laboratory, Germany

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Introduction

Examples of optical systems that use LED-sources (1)
- car-to-car communication based on VLC (figure from [1])
- LiFi-systems (figure from [2])
- VLC: Visible Light Communication
  - LED has limited modulation BW
  - time domain dispersion
  - ISI at high data rates

Explanation of Terms

Examples of optical systems that use LED-sources (2)
- plastic optical fiber transmission (figures from [3])
- BW limitation due to multipath dispersion
- Gaussian low-pass filter model
- ISI at high data rates

When does the term single carrier modulation make sense? (1)
- the figure shows the optical field for a coherent source
- we see a sinusoidal 'single carrier'
When does the term single carrier modulation make sense? (2)

- for coherent sources: $t_{\text{coherence}} \gg T_b$
- PSK or QAM can be used on a single or multiple carriers

For coherent sources, the term single carrier modulation makes sense because the coherence time is much larger than the bit duration, allowing coherent detection. For non-coherent sources, the optical field is broadband noise, not a single carrier!

Intensity Modulation and Direct Detection (IM/DD)

- we modulate the instantaneous optical power and use DD
- the optical signals are not described on the field-level, they are modeled on the instantaneous power level

\[ p_{\text{tx}}(t) = p_{\text{tx}}(t) \ast g(t) \]

- optical channel: LTI-system
- photodiode: responsivity $R$ in A/W
- noise: AWGN-current with power spectral density $N_0$

NRZ-PAM, example ($M = 4$)

- the figure shows the instantaneous optical power $p_{\text{tx}}(t)$
- we clearly see baseband pulses :-)

Basic Modulation Schemes for IM/DD
Basic Modulation Schemes for IM/DD

Single sub-carrier modulation: DC-biased QAM, example ($M = 4$)

A modulated electrical sub-carrier (a driving current) modulates the LED intensity: **sub-carrier modulation** [4, 5]

- Here we use **half wave rectifying** [5], also referred to as asymmetrically clipping (AC) [1]

Power spectral density examples

- DC-biased BPSK requires $2 \times$ more BW than OOK (for rectangular pulse-shaping)

- $P_{\text{OOK}}$: required optical power of OOK (reference)

- Do not use energy and bandwidth efficiency results from RF!

Required bandwidth and power for AWGN channels

- $P_{\text{OOK}}$: required optical power of OOK (reference)

- Do not use energy and bandwidth efficiency results from RF!
The Basic Schemes in Dispersive Channels

Gaussian low-pass filter: frequency response and impulse response

- valid model for a POF
- analytical calculations easily possible
- \( G(f) = e^{-\pi (ft)^2} \)  
- \( g(t) = \frac{1}{T} e^{-\pi (t/T)^2} \)  with  \( T = \sqrt{\frac{\ln 2}{\pi}} \)

Required average optical Rx-power (normalized)

\[ 10 \cdot \log_{10} \left( \frac{P_R}{\sqrt{N_0 R_b}} \right) \text{ dB} \]

relative data rate  \( R_b/f_{3opt} \)

NRZ-PAM,  \( M=2 \)

DC-QAM,  \( M=4 \)

Relative data rate  \( R_b/f_{3opt} \)

\( P_0 = 10^{-3} \)
**The Basic Schemes in Dispersive Channels**

### Required average optical Rx-power (normalized)

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Average Rx-power (normalized)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRZ-PAM, M=2</td>
<td>$P_{b} = 10^{-3}$</td>
</tr>
<tr>
<td>NRZ-PAM, M=4</td>
<td></td>
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<tr>
<td>NRZ-PAM, M=8</td>
<td></td>
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<tr>
<td>DC-QAM, M=4</td>
<td></td>
</tr>
<tr>
<td>AC-QAM, M=4</td>
<td></td>
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</tbody>
</table>

**NRZ-PAM: eye-diagram: $M = 2$, $R_b = 2f_{3\text{opt}}$**

- At Rx input

**DC-biased DMT:** we start with a normalized bipolar signal (1)

- **first:** time continuous signal without CP

$$x(t) = 2 \cdot \sum_{\mu=1}^{N_c} |Z_\mu| \cos \left(2\pi\mu \cdot \frac{t}{T} + \phi_\mu\right)$$

- $T$: symbol interval without CP (correlation interval at Rx)
- **same BW-efficiency** as M-PAM, if all $N_c$ sub-carriers are $M$-QAM modulated
- **note:** this signal is bipolar
- **this is a Fourier-series** with $Z_\mu$ (complex QAM-symbols) being the Fourier-coefficients

- **corresponding time discrete signal without CP with $t_0 = T/N$**

$$x[n] = 2 \cdot \sum_{\mu=1}^{N_c} |Z_\mu| \cos \left(2\pi\mu \cdot \frac{n}{N} + \phi_\mu\right), \quad n = 0, 1, \ldots, N - 1,$$
DC-biased DMT: we start with a normalized bipolar signal (1)

- the last equation can also be written as an IDFT:

\[
x[n] = \sum_{\mu=1}^{N_c} \left( Z_{\mu} e^{j2\pi\mu\cdot \frac{n}{N}} + Z_{\mu}^* e^{-j2\pi\mu\cdot \frac{n}{N}} \right)
\]

- in matrix-vector notation: \( x = F^H \cdot X \)

\[
X = \begin{bmatrix} 0 & Z_1 & Z_2 & \ldots & Z_{N_c} & 0 & 0 & \ldots & Z_{N_c-1} & \ldots & Z_1 \end{bmatrix}^T
\]

- \( F \) is a \( N \times N \) DFT-matrix; \( x = [x_0 \ x_1 \ \ldots \ x_{N-1}]^T \)

DMT: the purpose of cyclic prefix (CP)

- we consider a sub-carrier with \( \mu = 3 \)
- blue: transmitted signal without cyclic prefix

DMT: the purpose of cyclic prefix (CP)

- we consider a sub-carrier with \( \mu = 3 \)
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DC-biased DMT: biased optical signal and clipping (without CP)

- instantaneous optical power \( p[n] \) (discrete version):

\[
p[n] = \begin{cases} P_0 s[n] & s[n] \geq 0 \\ 0 & \text{else} \end{cases}, \quad \text{where} \]

\[
s[n] = x[n] + k_{\text{clip}} \sqrt{E\{x^2[n]\}}
\]

- \( x[n] \) is approximately Gaussian distributed with \( E\{x^2[n]\} = 0 \)
- \( E\{s[n]\} = \text{DC-bias} \)
- \( P = E\{p[n]\} \approx P_0 \cdot \text{DC-bias} \)
DMT: the purpose of cyclic prefix (CP)

- we consider a sub-carrier with $\mu = 3$
- red: received signal with cyclic prefix

A more general look at the cyclic prefix:

- we use nothing but frequency domain equalization:
  - within the correlation interval, the convolution of $p(t)$ with $g_{\text{tot}}(t)$ appears as a cyclic convolution:
    \[
    y(t) = p(t) * g_{\text{tot}}(t) = p(t) \hat{*} g_{\text{tot}}(t)
    \]
  - hence, for the discrete signals, we get:
    \[
    y[n] = p[n] \hat{*} g_{\text{tot}}[n] \quad \text{DFT} \quad Y[\mu] = P[\mu] \cdot G[\mu]
    \]
  - this works for any $p[n]$, not only for DMT!
- here we use zero-forcing equalization:
  \[
  Z_\mu \cdot P_0 \approx P[\mu] = Y[\mu] / G_{\text{tot}}[\mu], \quad \mu = 1 \ldots N_c
  \]
A more general look at the cyclic prefix:
- we use nothing but frequency domain equalization:
- within the correlation interval, the convolution of \( p(t) \) with \( g_{tot}(t) \) appears as a cyclic convolution:
  \[
  y(t) = p(t) \ast g_{tot}(t) = p(t) \widehat{\ast} g_{tot}(t)
  \]
- hence, for the discrete signals, we get:
  \[
  y[n] = p[n] \ast g_{tot}[n] \Rightarrow Y[\mu] = P[\mu] \cdot G[\mu]
  \]
  \( \Rightarrow \text{this works for any } p[n], \text{ not only for DMT!} \)
- here we use zero-forcing equalization:
  \[
  Z_{\mu} \cdot P_0 \approx P[\mu] = \frac{Y[\mu]}{G_{tot}[\mu]}, \mu = 1 \ldots N_c
  \]

Required average optical Rx-power for DC-biased DMT
\[
P_b = 10^{-3}, \text{ no power loading (red)}
\]

\[
\frac{10 \cdot \log_{10} \left( \frac{PR}{\sqrt{N_0 R_b}} \right)}{\text{dB}} \quad \text{relative data rate } R_b/f_{3opt}
\]
AC-DMT: we use only the odd sub-carriers
- no clipping noise in odd sub-carriers after half wave rectifying

Required average optical Rx-power for AC-biased DMT

$$P_b = 10^{-3}, M_{\text{max}} = 1024$$

$$10 \log_{10} \left( \frac{PR}{\sqrt{N_0 R_b}} \right) \text{dB}$$

relative data rate $R_b/f_{3\text{opt}}$

NRZ-OOK (no equalization)
- $M=4$
- $M=16$
- $M=64$
- $M=256$
- $M=1024$

optimal bit-loading
- $P_b = 10^{-3}, M_{\text{max}} = 1024$

DC-biased DMT, optimal BL
**Advanced Transmission Schemes with FDE: PAM**

### Signal structure and Rx-block diagram for CP-usage
- The FFT-usage can be enabled by means of a CP.
- ADC ➔ remove CP / FFT ➔ ZF-equalization (point-wise mult.) ➔ IFFT

--sync channel estimation

<table>
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<tr>
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<th>CP 2</th>
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### Signal structure and Rx-block diagram for unique word usage
- But it's also possible to use a unique word (UW), see [2].
- Here, the FFT-window needs to include the UW part.

- ADC ➔ FFT ➔ ZF-equalization (point-wise mult.) ➔ IFFT

- Sync channel estimation

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### Why PAM-block transmission as an alternative? (1)
- For low-pass channels, power efficient PAM is a natural choice.
- Disadvantages of DMT:
  - Bit-loading needs full channel knowledge at Tx
  - High peak to average power ratio ⇒ power hungry drivers
  - DC-biased DMT requires a very careful adjustment of the bias
  - Bit-loading enhanced AC-DMT can hardly combined with DC-balance
- Advantages of PAM:
  - Simple and energy efficient LED drivers
  - Adaptive Tx needs only modulation order
    (Note: larger symbol duration by means of repetition coding)
  - Simple DC-balance by XBYM line-codes (X Bits are mapped to Y M-level PAM-symbols)

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### Why PAM-block transmission as an alternative? (2)
- If $M - 1$ LEDs are used for PAM, we only need binary drivers.
- 5 level PAM example
- 5-PAM can be combined with a 8B4P line code to ensure DC balance.
**Advanced Transmission Schemes with FDE: PAM**

**Required average optical Rx-power for PAM**

\[
P_b = 10^{-3}, \text{NRZ-rectangular pulses, unique word}
\]

![Graph](image)

**Advanced Transmission Schemes: PAM with FDE and DFE**

**Receiver structure**

- \(\text{ADC} \ (T/2)\)
- \(\text{FFT} \ (N)\)
- \(\text{lin. equalization} \ (\text{and 2-fold dec.})\)
- \(\text{IFFT} \ (N/2)\)
- \(\text{DFE} \ (T)\)

- The linear frequency domain equalizer acts as a phase equalizer
- It realizes a **sample whitened matched filter** for the received pulses
- The equivalent discrete system between the transmitter and DFE-input (DFE: decision feedback equalizer) is a **minimum phase system**
Conclusions

- do not use energy and bandwidth efficiency results from RF
- a bandwidth efficient modulation scheme does not necessarily provide a good performance in dispersive channels
- to benefit from bandwidth efficient modulation, equalization is required
- here, frequency domain equalization has been considered for DMT and PAM
  - PAM shows very good performance even for linear equalization
  - FDE plus DFE gives best performance
- Outlook:
  - a very reliable synchronization and channel estimation approach has been presented in [3]
  - mutual information rates of PAM and DMT will be analyzed in our Icton 2017 contribution

Literature (1)

