**dB/dBW/dBm cheat sheet**

- **dB** is a logarithmic ratio of powers. So saying $P_1 = 2 \cdot P_2$ is equivalent to saying $P_1 / P_2 \approx 3 \text{ dB}$, because

$$10 \cdot \log_{10} \left( \frac{P_1}{P_2} \right) \text{ dB} = 10 \cdot \log_{10}(2) \text{ dB} = 3 \text{ dB}.$$ 

The unit dBi can be treated just like dB, because it measures the gain of an antenna relative to an isotropic antenna which has the gain of 1 (0 dB).

- **dBm and dBW** is a (logarithmic) unit to measure powers. Since dB is a ratio of powers, dBm and dBW are defined by forming the ratio of the power you want to express relative to a reference power, which is 1 W for dBW and 1 mW for dBm. Formally speaking,

$$P_T |_{\text{dBm}} = 10 \cdot \log_{10} \left( \frac{P_T}{1 \text{ mW}} \right)$$

$$P_T |_{\text{dBW}} = 10 \cdot \log_{10} \left( \frac{P_T}{1 \text{ W}} \right) = P_T |_{\text{dBm}} - 30 \text{ dB}$$

Consequently, two things are okay:

- Starting with a power (dBW or dBm) you can add and subtract ratios (dB or dBi) as often as you like and you still have a power (dBW or dBm):

$$P_1 \cdot G_1 / L_1 = P_2 \iff 10 \cdot \log_{10} \left( \frac{P_1 \cdot G_1 / L_1}{1 \text{ W}} \right) = 10 \cdot \log_{10} \frac{P_2}{1 \text{ W}}$$

$$\iff P_1 |_{\text{dBW}} + G_1 |_{\text{dB}} - L_1 |_{\text{dB}} = P_2 |_{\text{dBW}}$$

- **Subtracting two powers** (dBW or dBm) which is equivalent to computing their ratio (dB):

$$\frac{P_1}{P_2} \approx 10 \cdot \log_{10} \left( \frac{P_1}{P_2} \right) \text{ dB} = 10 \cdot \log_{10} \left( \frac{P_1}{1 \text{ W}} \right) - 10 \cdot \log_{10} \left( \frac{P_2}{1 \text{ mW}} \right) = P_1 |_{\text{dBW}} - P_2 |_{\text{dBW}}$$

$$= 10 \cdot \log_{10} \left( \frac{P_1}{1 \text{ mW}} \right) - 10 \cdot \log_{10} \left( \frac{P_2}{1 \text{ mW}} \right) = P_1 |_{\text{dBm}} - P_2 |_{\text{dBm}}$$

Both powers must have the same unit, do not mix dBW and dBm.

**Short hand notation for things that are okay:**

<table>
<thead>
<tr>
<th>dBW ± dB</th>
<th>dBW</th>
</tr>
</thead>
<tbody>
<tr>
<td>dBm ± dB</td>
<td>dBm</td>
</tr>
<tr>
<td>dBW − dBW</td>
<td>dB</td>
</tr>
<tr>
<td>dBm − dBm</td>
<td>dB</td>
</tr>
</tbody>
</table>
On the other hand, the following things are not okay:

- **Never ever multiply dBW with dB!** I have seen students claiming that a power of $P_T = 10 \text{ W}$ with an antenna gain of $G_T = 10$ gives an effective radiated power of 100 dBW. **No!** 100 dBW is 10 Gigawatts! Multiply in linear scale, that becomes addition in logarithmic scale:

  $$P_T \cdot G_T = 10 \text{ W} \cdot 10 = 100 \text{ W} \triangleq 20 \text{ dBW}$$

  $$\iff P_T|_{\text{dBW}} + G_T|_{\text{dBi}} = 10 \text{ dBW} + 10 \text{ dBi} = 20 \text{ dBW}$$

- **Never add a bunch of quantities in dBW or dBm!** Adding powers in log scale means multiplying them in linear scale. If you add 10 dBW with 3 dBW and 6 dBW then you have

  $$10 \text{ dBW} + 3 \text{ dBW} + 6 \text{ dBW} \triangleq 10 \text{ W} \cdot 2 \text{ W} \cdot 4 \text{ W} = 80 \text{ W}^3$$

  What unit is Watt cubed? I don’t know, you tell me...

- A power cannot be measured in dB. A gain/loss cannot be measured in dBm or dBW.