PAM-transmission with optimal detection for dispersive optical channels with intensity modulation and direct detection

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ABSTRACT

Digital transmission over dispersive optical channels suffers from intersymbol interference (ISI). We study the mutual information of PAM transmission under the intensity modulation and direct detection (IM/DD) constraint, where we model the optical channel as a Gaussian low-pass filter, which is a valid attempt for polymer optical fibers (POF) and allows closed form expressions. In particular, we study the properties of the sample whitened matched filter, which enables the derivation of the mutual information for PAM with IM/DD. The information rate (lower bound) of PAM with equally probable input symbols is compared to an information rate approximation of DC biased multiple sub-carrier modulation (MSM), where we approximate the effect due to threshold clipping as Gaussian distributed noise, and to the information rate of asymmetrically clipped MSM. The required optical power for uncoded block transmission at a bit error rate of $10^{-3}$ is also presented. In the case of PAM, a unique word approach is used, and decision-feedback equalization is performed directly at the output of a sample whitened matched filter realized in the frequency domain. The results show that PAM with a rectangular pulse shape outperforms the other schemes.

Keywords: channel capacity, mutual information, information rates, intensity modulation, polymer optical fiber (POF) communication, wireless optical communication, orthogonal frequency division multiplex (OFDM), discrete multitone transmission (DMT), pulse-amplitude modulation (PAM), multiple sub-carrier modulation

1 INTRODUCTION

In this paper, we discuss digital transmission over dispersive optical channels with IM/DD. Typical examples are indoor free space optical channels or polymer optical fiber (POF) channels. Due to the dispersive nature of these channels, the transmission suffers from ISI. In order to combat these effects, advanced modulation and equalization schemes are required.

Discrete multitone transmission\(^1\) (DMT), which relies on multiple sub-carrier modulation (MSM), is a promising candidate [1]. Here, the LED or Laser diode driving current is modulated by means of DC-biased or half-wave rectified QAM-modulated electrical sub-carriers. The biasing or half-wave rectifying is required in order to ensure the non-negative constraint of intensity modulation. In [2, 3], we have shown that PAM block-transmission offers an at least equivalent performance, even if only linear zero-forcing equalization is used. PAM block transmission is characterized by an unique word (or cyclic prefix) insertion into the data stream [4], very similar to DMT/OFDM. This insertion does not only prevent inter-block interference even in dispersive channels, but also enables the usage of the discrete Fourier transform (DFT) and a subsequent frequency-domain signal processing at the receiver, since the received signal appears as a cyclic signal within the DFT window.

However, PAM does not show the best performance, if linear equalization is used. Therefore we discuss the properties of the optimal sample whitened matched filter in section 3.1. Moreover, we develop the expression of the lower bound of the PAM information rate, i.e., the maximum error free data rate which is offered by the optical ISI-channel under the IM/DD constraint. The optical channel is modeled as a Gaussian low-pass filter, which is a valid model for POF transmission, but the adaption of the results to other channel models is straightforward. For general linear ISI channels with additive Gaussian noise, which allow the transmission of zero mean bipolar signals $s(t)$, serial transmission is capacity approaching under the second moment constraint of $s(t)$, if a sample whitened matched filter is used [5]. To approach capacity, maximum-likelihood sequence estimation combined with powerful coding may be performed directly at the sample whitened matched filter output. In this paper, however, we discuss the mutual information/ the information rate of PAM under the average optical power constraint, which is the first moment of the signal, not the second moment.

In the sections 4.2 and 4.3, we derive the information rates for MSM and compare the results in section 5. As for PAM, we do not consider any overhead due to a cyclic prefix or a unique word, if we discuss the maximum information rates. Thus the channel spacing is assumed to be infinitely small, and the well known integral expression for the channel capacity of channels with a second moment constraint is used as a basis. This helps us to estimate the information rates of MSM in IM/DD channels with the first moment constraint. In [6], similar investigations for DC-biased MSM have been presented, and the results are claimed to show the ultimate capacity of SI-POF links. However, the impact of threshold clipping has not been considered at all. Moreover, the theoretical capacity of a channel itself cannot depend on signal parameters (the relative Bias and the modulation index, respectively).

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\(^1\)We use the term DMT and not OFDM to make clear that the IDFT output signal at the transmitter is real valued for IM/DD. In the opposite to systems with field modulation, no quadrature up-conversion takes place with that signal.
Just as we ignore the cyclic prefix or unique word overhead and assume an infinitely small sub-carrier spacing for MSM, which corresponds to an infinitely high DFT-length of DC-biased or asymmetrically clipped (AC) DMT, we ignore the realization aspects of the sample whitened matched filter in section 3.1. However, in practical PAM block transmission systems with frequency domain equalization, the linear filtering can be completely realized in the frequency domain. We discuss this briefly in section 5 and compare the performance of an uncoded block transmission PAM system with fractional sampling, filtering in the frequency domain and decision-feedback equalization in the time domain, with those of DC-biased DMT and AC-DMT.

2 CHANNEL MODELLING

The optical channel is modeled as a linear system with respect to the instantaneous optical power. The photodiode exhibits a responsivity $R$ (in A/W) and maps the received optical power into a current. The dispersive channel is modeled as a Gaussian low-pass filter. The noise current $n(t)$ is assumed to be signal independent and modeled as additive white Gaussian noise (AWGN) with a one-sided power spectral density (p.s.d.) $N_0$ (in $A^2/Hz$). Our figure of merit is the required average optical power $P = E[(p(t))]$ at the transmitter (Tx), i.e., at the LED or laser diode output. Thus, instead of using the required $E_{b,elec}/N_0$ ratio as a performance measure, where $E_{b,elec}$ is the electrical energy per bit with $E_{b,elec} \sim (PR)^2/R_b$, we use the required power ratio $PR/\sqrt{R_bN_0}$.

\[
\begin{array}{c}
\text{(a)} & p(t) \xrightarrow{g_C(t)} R \xrightarrow{n(t)} \text{(b)} & p(t) \xrightarrow{g_C(t)} R \xrightarrow{n(t)}
\end{array}
\]

Figure 1. Channel modeling. Although the photodiode is located at the receiver, the output signals in (a) and (b) are same.

3 MAXIMUM INFORMATION RATE OF PAM-TRANSMISSION

3.1 Transmission Model and Optimal Detection

Our system model is shown in Fig. 2(a). All signals are real valued. Pulse shaping is achieved by means of a unit energy Tx-filter. If a NRZ (non-return-to-zero) rectangular pulse shape is assumed, the impulse response $g_T(t)$ of this filter is given as

\[
g_T(t) = \frac{1}{\sqrt{T}} \cdot \text{rect}(t/T),
\]

where $T$ denotes the symbol interval. The data symbols $z_k$ to be transmitted are taken from an unipolar alphabet, i.e., $z_k \in \{0, 1, \ldots, M-1\}$, where $M$ is the modulation order. For each symbol $z_k$ and the pulse-shape defined in (1), the corresponding (one-dimensional) signal vector $x_k$ is given as

\[
x_k = R \cdot P_0 \cdot \sqrt{T} \cdot z_k \quad \text{with } P_0 = \frac{2P}{M-1},
\]

where $P$ defines the average optical power. Clearly, the signal at the output of $g_T(t)$ is non-negative and satisfies the IM/DD constraint.

The dispersive optical channel is modeled as a Gaussian low-pass filter with an optical 3 dB cut-off frequency $f_{3,\text{opt}}$, which corresponds to an electrical 6 dB cut-off frequency. Its transfer function is given as

\[
G_C(f) = e^{-\frac{\ln(2) \cdot f^2}{f_{3,\text{opt}}^2}}.
\]

With respect to the Rx-filter, we assume a sample-whitened matched filter, which provides a sufficient statistic for optimal detection, see [5, 7]. This filter consists of a continuous-time matched filter for the received pulses $g_{rx}(t)$, i.e.,

\[
g_{R1}(t) = g_{rx}(-t) \quad \text{with } g_{rx}(t) = g_T(t) \ast g_C(t),
\]

and a discrete-time filter with impulse response $g_{R2}[k]$. The discrete filter ensures white noise at the sampling times and maps the discrete impulse response between the TX-symbols $x_k$ and the RX-symbols $y_k$ into a causal minimum-phase characteristic. In order to obtain $g_{R2}[k]$ or its $z$-transform $g_{R2}[z] \triangleq G_{e,R2}(z)$, it is necessary to decompose the discrete impulse response $q[k]$ between the transmitter and the sampled matched filter output into a monic and causal minimum-phase part $g_{\text{min}}[k]$ and a purely anti-causal part $g_{\text{anti}}[k] = g_{\text{min}}[-k]$. The impulse response $q[k]$ is equal to the sampled autocorrelation function $\varphi_{g_{rx}g_{rx}}(\tau)$ of $g_{rx}(t)$ and we get

\[
q[k] = \varphi_{g_{rx}g_{rx}}(kT) = A^2 \cdot g_{\text{min}}[k] \ast g_{\text{anti}}[k] = g_{\text{min}}[1] = g_{\text{anti}}[1] = 1.
\]
The spectral factorization of
\[ Q_s(e^{j2\pi fT}) = P[|G_{rx}(f)|^2]_{f_p=1/T} = A^2 \cdot G_{z,\text{min}}(e^{j2\pi fT}) \cdot G_{z,\text{anti}}(e^{j2\pi fT}), \]  
(6)
where \( q[k] \sim Q_s(z) \), may be obtained by expressing the cepstrum of the folded spectrum \( P[|G_{rx}(f)|^2]_{f_p=1/T} \) as a Fourier series \([5]\). The factor
\[ A^2 = \exp \left( T \int_{1/T} \ln Q_s(e^{j2\pi fT}) df \right) \]
(7)
is the geometric mean of the real-valued transfer function \( Q_s(e^{j2\pi fT}) \). For a discrete filter with a transfer function
\[ G_{z,R2}(z) = \frac{1}{A^2} \cdot \frac{1}{G_{z,\text{anti}}(z)} = \frac{1}{A^2} \cdot \frac{1}{G_{z,\text{ant}}(1/z)}. \]
(8)
see Fig. 2(a), the channel between \( x[k] \) and \( y[k] \) equals \( g_{\text{min}}[k] \), and the noise samples are white with a second moment of
\[ \frac{N_0}{2} \cdot \frac{P[|G_{rx}(f)|^2]_{f_p=1/T}}{Q_s(e^{j2\pi fT})} = \frac{1}{A^2} \cdot \frac{1}{|G_{z,\text{anti}}(e^{j2\pi fT})|^2} = \frac{N_0}{2}/A^2. \]
(9)
Thus, the unit-less factor \( 1/A^2 \), with \( A^2 \leq 1 \), is a direct measure of the noise enhancement. For an ideal AWGN-channel with \( G_C(f) = 1 \), we get \( g_{\text{min}}[k] = \delta[k] \), \( A^2 = 1 \) and thus a noise variance of \( N_0/2 \).

\[ \begin{align*}
\text{(a)} & \quad \text{sample whitened matched filter} \\
\text{(b)} & \quad \text{NRZ-PAM (rectangular pulses)} \\
\end{align*} \]

\[ \begin{align*}
&\text{Figure 2. Equivalent discrete-time system model for optical PAM transmission (a); Maximum normalized data rate} \\
&\text{(lower bound) as a function of the normalized optical power for NRZ-PAM based on rectangular pulses (b).} \\
&\end{align*} \]

### 3.2 Lower Bounds of Mutual Information and Information Rate

For a sample-whitened matched filter, the lower bound \( I_L(X;Y) \) (in [bits]) of the mutual information between the channel input and output is given as \([7]\)
\[ I_L(X;Y) = I(X;X \cdot A + N), \]
(10)
where \( N \) is zero mean Gaussian noise with a variance of \( N_0/2 \) and \( X \) is a i.i.d. random variable, where
\[ X \in \{0, R P_0 \sqrt{T}, \ldots, (M-1) R P_0 \sqrt{T}\}, \]
if the NRZ-rectangular pulse shape is used. The lower bound can be interpreted as the average mutual information that corresponds to an ideal decision feedback equalizer with error-less past decisions, see \([7]\), which processes the samples \( y_k \). Thus the lower bound of the maximum information rate is given as \( R_0 = I_L(X;Y)/T \) (in bits/s).

**Example:** For a pulse-shape according to (1) and OOK (On-Off Keying), we get
\[ I_L(X;Y) = H(X \cdot A + N) - H(N) \text{ with } H(N) = \frac{1}{2} \log_2(\pi e N_0) \text{ and } H(Y) = \int_{-\infty}^{\infty} f_Y(y) \log_2(f_Y(y)) dy, \]
where we assume i.i.d. input symbols and thus

$$f_Y(y) = \frac{1}{2} \cdot \frac{1}{\sqrt{\pi N_0}} \left( e^{-y^2/N_0^2} + e^{-(y-A^2PR\sqrt{T})^2/N_0^2} \right).$$

### 3.3 Discussion

The results for NRZ-PAM with a fixed modulation order are shown in Fig. 2(b). Both green curves correspond to a fixed symbol rate, namely $1/T = f_{3, opt}$ and $1/T = 3f_{3, opt}$, and OOK. Clearly, for OOK, $I(X; Y) \leq 1$. Thus the maximum normalized data rates $R_b/f_{3, opt}$ are equal to 1 and 3. If the normalized optical power $PR/\sqrt{R_b N_0}$ is lower than 2.5 dB, $1/T = f_{3, opt}$ is a better choice than $1/T = 3f_{3, opt}$, since the transmission suffers from less dispersion and benefits form a larger value of $A$.

All the other results shown in the figure correspond to symbol rates which maximize the information rate $R_b$ at a given optical power. OOK is the best choice for normalized optical powers of less than 5 dB, whereas a 4-PAM is optimal between 5 and 7.6 dB, and a 16-PAM above 11 dB. Although we are mainly interested in values $R_b > f_{3, opt}$ where the channel introduces dispersion, it is worth to discuss the lowest optical power level, which is about $-0.79$ dB. Since the theoretical $E_{b, elec}/N_0$ limit of unipolar OOK-transmission is limited to $2 \cdot \ln(2)$ (or 1.42 dB), and the average electrical power $E_{b, elec}$ corresponds to $E_{b, elec} = 2(PR)^2/R_b$ for the NRZ-rectangular pulse shape as defined in (1), the lower limit of $PR/\sqrt{R_b N_0}$ equals $\sqrt{\ln 2} = \ln(1)$, corresponding to $-0.79$ dB.

### 4 MAXIMUM INFORMATIONS RATES FOR MULTIPLE SUB-CARRIER MODULATION

#### 4.1 Capacity of a Linear Gaussian Channel without IM/DD constraint

The capacity of a linear Gaussian channel with bipolar inputs may be approached both by serial transmission and by multi-carrier transmission, see [5]. In this section, we ignore the constraint of non-negative signals and consider the transmission of a multi-carrier signal $s_{bip}(t)$ over a channel according to Fig. 3(a). All signals are still real valued and represent currents. The filter transfer function is defined in (3), where we may replace $f_{3, opt}$ by its electrical equivalent $f_{b, elec}$. Based on the capacity analyses of this section, we estimate the maximum information rates of optical MSM-schemes in section 4.2 and 4.3 and compare the results with those of PAM.

If we assume an average power constraint of $s_{bip}(t)$, i.e., $E\{s_{bip}(t)^2\} = m_{bip}^{(2)}$, the channel capacity $C$ (in bits/s) of the channel shown in Fig. 3(a) is given as [5]

$$C = \int_0^B \log_2 \left( 1 + \frac{\Phi_{bip}(f)}{N_0/|GC(f)|^2} \right) \, df. \quad (11)$$

The term $\Phi_{bip}(f)$ denotes the one-sided p.s.d. of $s_{bip}(t)$, where the largest mutual information is achieved by the water pouring solution [5]

$$\Phi_{bip}(f) = \begin{cases} K - \frac{N_0}{|GC(f)|^2} & 0 \leq f \leq B \\ 0 & f > B \end{cases}. \quad (12)$$

For the Gaussian low-pass channel according to (3), the transmission band stops at a frequency

$$B = f_{3, opt} = f_{b, elec} \cdot \sqrt{\frac{\ln(K/N_0)}{2 \ln 2}}, \quad (13)$$

since $B$ must satisfy the property $K = N_0/|GC(B)|^2$, i.e., $\Phi_{bip}(f) > 0$. Thus, with increasing $B$, the water-level $K$ increases as well, and the average electrical power $m_{bip}^{(2)}$ is given as

$$m_{bip}^{(2)} = \int_0^B \Phi_{bip}(f) \, df = K \cdot B - \frac{N_0}{2} \cdot B \cdot \sqrt{\frac{\pi}{\ln(K/N_0)}} \cdot \text{erfi} \left( \sqrt{\ln(K/N_0)} \right), \quad (14)$$

where $\text{erfi}(\cdot)$ denotes the imaginary error function. The results shown in Fig. 3(b) have been obtained by varying $K$, where $K > N_0$. The values of $B$, $\Phi_{bip}(f)$ and $m_{bip}^{(2)}$ have then been calculated using (12), (13) and (14). The electrical energy per bit $E_{b, elec}$ is given as $E_{b, elec} = m_{bip}^{(2)}/C$. The figure nicely shows the minimum required $E_{b, elec}/N_0$ of $-1.59$ dB.
thus the relation between

In the previous section, the constraint of non-negative signals was not considered. The distribution of \( s_{\text{bip}}(t) \) tends to be Gaussian with zero mean, since it consists of many independent components. Thus an optical Tx-signal \( p(t) \) with \( p(t) \geq 0 \) will never appear exactly as \( s_{\text{bip}}(t) + g_c(t) \) at the photodiode output, at most its AC-component may be identical if \( s_{\text{bip}}(t) \) is accordingly biased. Therefore we discuss information rates and avoid the term channel capacity in the following.

One solution to satisfy the IM/DD-constraint is DC-biasing combined with threshold clipping. In this case, the optical signal \( p(t) \) at the transmitter is given as

\[
    p(t) = \begin{cases} 
    1/R \cdot s_{\text{bias}}(t) & s_{\text{bias}}(t) \geq 0 \\
    0 & \text{else}
    \end{cases}
\]

with \( s_{\text{bias}}(t) = s_{\text{bip}}(t) + \mu_{\text{DC}} \cdot \sqrt{m_{\text{bias}}^{(2)}} \).

The signal \( s_{\text{bias}}(t) \) is the biased version \( s_{\text{bip}}(t) \), where the bias is chosen depending on the standard derivation of \( s_{\text{bip}}(t) \). The term \( \mu_{\text{DC}} \) is the relative bias and determines the amount of clipping and the mean optical power. At \( \mu_{\text{DC}} = 1.5 \), the clipping probability is 6.62 %. We restrict our attention to values \( \mu_{\text{DC}} \geq 1.5 \), which ensures that

\[
    P = \mathbb{E}\{p(t)\} \approx \mu_{\text{DC}} \cdot \sqrt{m_{\text{bias}}^{(2)}/R},
\]

and consider the AC-component of the clipped signal components

\[
    c(t) = R \cdot p(t) - s_{\text{bias}}(t)
\]
as additional, signal independent Gaussian noise which is not considered for water pouring. So (12), which only depends on \( N_0, |g_c(f)|^2 \) and \( m_{\text{bias}}^{(2)} \), is still used to determine the p.s.d. of the AC signal component \( s_{\text{bip}}(t) \), where the relation between \( m_{\text{bip}}^{(2)} \) and the average optical power is given by (16). If \( \Phi_{cc}(f) \) denotes the in-band (\( f \leq B \)) p.s.d. of the clipping noise \( c(t) \) (AC-component only), the estimate of the maximum theoretical information rate is thus

\[
    R_b \approx \int_0^B \log_2 \left( 1 + \frac{\Phi_{\text{bip}}(f)}{\Phi_{cc}(f) + N_0/|g_c(f)|^2} \right) \, df.
\]

In [8], the clipping is ignored at all. However, as shown in Fig. 4(b), this assumption is only valid for \( \mu_{\text{DC}} \geq 4 \), if the relative optical power is \( \leq 30 \) dB. At lower \( \mu_{\text{DC}} \) values, the maximum information rate is overestimated, if \( c(t) \) is ignored. In Fig. 4(b), this is shown for \( \mu_{\text{DC}} = 1.5 \). In [9], a white p.s.d. is assumed for \( c(t) \). Since threshold clipping is a non-linear process, even this assumption is not appropriate. This is shown in Fig. 4(a) for a MSM-signal with 1024 independent carriers. Overall it can be concluded that the relative proportion of the out of band interference increases with \( \mu_{\text{DC}} \). Thus, if the variance of \( c(t) \) is denoted as \( \sigma^2_c \), \( \Phi_{cc}(f) \) is generally \( < \sigma^2_c / B \).

Whereas the clipping probability decreases with a larger relative bias, the absolute amount of clipping increases with \( P \) for a given \( \mu_{\text{DC}} \). So \( \mu_{\text{DC}} \) must be chosen carefully. For a (very large) relative optical power of \( 30 \) dB, the best result has been obtained for \( \mu_{\text{DC}} = 4 \).
not suffer from clipping noise, but from halved signal amplitudes. This means that a signal can be assigned to an optical power of \( s_{\text{opt}} \) for a mean electrical power according to (14), and deactivate every second carrier afterwards (e.g., the ones with an even index), where we may denote this new multi-carrier signal as \( s_{\text{II}}(t) \), the maximum information rate of \( s_{\text{II}}(t) \) is clearly

\[
R_b = \frac{1}{2} \int_0^B \log_2 \left( 1 + \frac{\Phi_{\text{bip}}(f)}{N_0/G_C(f)^2} \right) \, df,
\]

if we still use the p.s.d. expression of \( s_{\text{bip}}(t) \) given in (12). The corresponding average power of \( s_{\text{II}}(t) \) is also halved compared to \( s_{\text{bip}}(t) \), and reads

\[
m_s^{(2)} = \frac{1}{2} \int_0^B \Phi_{\text{bip}}(f) \, df = \frac{1}{2} \left( K \cdot B - \frac{N_0}{2} \cdot B \cdot \sqrt{\frac{\pi}{\ln(K/N_0)}} \right) \cdot \text{erf} \left( \frac{\sqrt{\ln(K/N_0)}}{\sqrt{\ln(K/N_0)}} \right).
\]

If we now introduce half-wave rectifying on \( s_{\text{II}}(t) \) as a final step, the corresponding correlator output signals would not suffer from clipping noise, but from halved signal amplitudes. This means that a signal 2\( s_{\text{II}}(t) \), after it has been half-wave rectified, yields to the same mutual information as an unclipped signal \( s_{\text{II}}(t) \). Thus an instantaneous optical power of

\[
p(t) = \begin{cases} 
1/R \cdot 2 \cdot s_{\text{II}}(t) & s_{\text{II}}(t) \geq 0 \\
0 & \text{else}
\end{cases}
\]

can be assigned to \( s_{\text{II}}(t) \) which gives an average optical power of

\[
P = \mathbb{E}\{p(t)\} = \frac{1}{R} \int_0^1 4 \cdot k^{(2)} \, \, dt.
\]

To obtain numerical results, we start again with \( K \), calculate \( B \) according to (13), \( \Phi_{\text{bip}}(f) \) according to (12), and calculate \( m_s^{(2)} \) and \( R_b \) with (19) and (18).

### 4.3 Information Rate of Asymmetrically Clipped MSM

An optical MSM-signal can be also be obtained by means of half wave rectifying [1, 10], where the term asymmetrically-clipped DMT (or OFDM) is often used to describe practical, cyclic prefix based implementations. In the case of half wave rectifying, only half of the sub-carriers in \( s_{\text{bip}}(t) \) are used.

If we initially transmit with a bipolar multi-carrier signal \( s_{\text{bip}}(t) \) according to section 4.1, which achieves a capacity \( C \) for a mean electrical power according to (14), and deactivate every second carrier afterwards (e.g., the ones with an even index), where we may denote this new multi-carrier signal as \( s_{\text{II}}(t) \), the maximum information rate of \( s_{\text{II}}(t) \) is clearly

\[
R_b = \frac{1}{2} \int_0^B \log_2 \left( 1 + \frac{\Phi_{\text{bip}}(f)}{N_0/G_C(f)^2} \right) \, df,
\]

if we still use the p.s.d. expression of \( s_{\text{bip}}(t) \) given in (12). The corresponding average power of \( s_{\text{II}}(t) \) is also halved compared to \( s_{\text{bip}}(t) \), and reads

\[
m_s^{(2)} = \frac{1}{2} \int_0^B \Phi_{\text{bip}}(f) \, df = \frac{1}{2} \left( K \cdot B - \frac{N_0}{2} \cdot B \cdot \sqrt{\frac{\pi}{\ln(K/N_0)}} \right) \cdot \text{erf} \left( \frac{\sqrt{\ln(K/N_0)}}{\sqrt{\ln(K/N_0)}} \right).
\]

Figure 4. Power spectral density of the AC-component of \( c(t) \) (a). Normalized information rate of DC-MSM (b).

### 5 COMPARISON

Fig. 5(a) shows the information rates for DC-MSM, AC-MSM and PAM, where we have also considered 50 % RZ-PAM based on rectangular pulses. In contrast to Fig. 2(b), an optimal PAM modulation order has been chosen

\[2\text{It is necessary that each sub-carrier } s_{\text{II}, \mu}(t) \text{ of } s_{\text{II}}(t) \text{ exhibits the same half-wave symmetry within the correlation interval } T, \text{ i.e., an odd symmetry with } s_{\text{II}, \mu}(t) = -s_{\text{II}, \mu}(t - T/2) \text{ for } 0 \leq t \leq T/2.\]

\[3\text{However, } M \text{ has been chosen only as a power of 2.}\]
for each $P$. The 1.5 dB gain of RZ-PAM at very low data rates is a result of the photodiode characteristic, which converts an optical power into a current, see [11]. Thus, even if NRZ-OOK and RZ-OOK require the same electrical energy per bit, the corresponding optical powers differ. Fig. 5(a) shows that the maximum information rates of all the schemes are similar, where DC-MSM outperforms AC-MSM for $R_b/f_{3,opt} > 6$. However, PAM, which is a natural choice for low-pass channels, performs best.

While the information theoretic limits shown in Fig. 5(a) assume powerful coding, the results in Fig. 5(b) show the required optical power for uncoded transmission at a bit error rate (BER) of $10^{-3}$. With respect to PAM, block transmission with a unique word insertion [4] is assumed. The received signal is fractionally sampled with a rate $2/T$ and further processed in the frequency domain (DFT-size 512). This includes the implementation of an approximated version of the channel matched filter $G_{R1}(f)$ shown in Fig. 2(a), and a frequency domain implementation of the whitening filter $G_{z,R2}(e^{j2\pi fT})$. Since $G_{z,R2}[k]$ operates with the symbol clock, a factor 2 spectral folding is done prior to $G_{z,R2}(e^{j2\pi fT})$. As a last step, decision-feedback equalization (DFE) is performed on the time domain signal, which is obtained with an IDFT of size 256. With every new block, the 12 DFE registers are initialized with the exact unique word content to avoid a feedback of erroneous past decisions. For DMT, the same DFT-size is assumed. Here, optimal bit and power loading is used [12], where the maximum modulation order $M$ was set to 4096 in both cases. The redundancy due to the cyclic prefix (DMT) or the unique word (PAM) is approximately 11%. The results show once more the very good properties of PAM.

6 CONCLUSION

We have derived the information theoretic limits of PAM and MSM transmission in dispersive optical channels with IM/DD. The comparison of PAM, DC-biased MSM, and asymmetrically clipped MSM reveals a similar performance for channels with a Gaussian low-pass characteristic, where PAM shows the best results. A key element to obtain the optimal PAM performance is the sample whitened matched filter. This filter maps the discrete impulse response between the transmitted and received symbols into a minimum phase characteristic. For PAM block transmission with fractional sampling, this filter can be approximatively implemented in the frequency domain. A decision-feedback equalizer, which further processes the filtered signal in the time domain, benefits from a periodic reset of the feedback registers by means of the unique word, which frames every PAM block. Even for uncoded systems, which operate at a bit error rate of $10^{-3}$, PAM performs best.

REFERENCES


